On the Relevance of Catenary-Based Models for Underwater Tethered Robots
An experimental study

Journée GT2 Robotique Marine et Sous-marine

Martin Filliung

Laboratoire d'informatique et systèmes — Nicolas Boizot
Laboratoire de conception de systèmes mécaniques et robotiques — Claire Dune
Université de Toulon

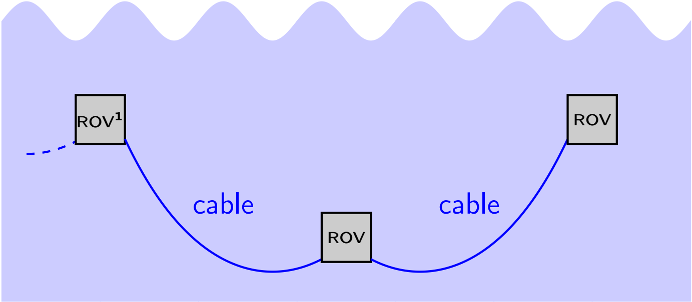
December 15, 2023

Outline

1. Introduction
2. Model, residual and parameter estimation
3. Experiments

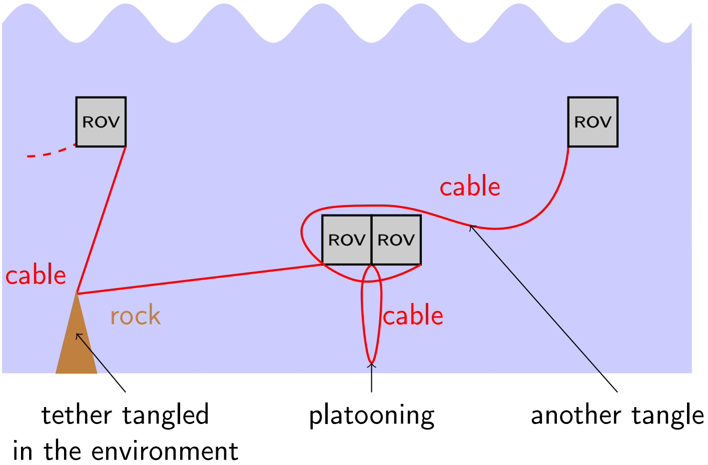
Introduction

Underwater robot chain

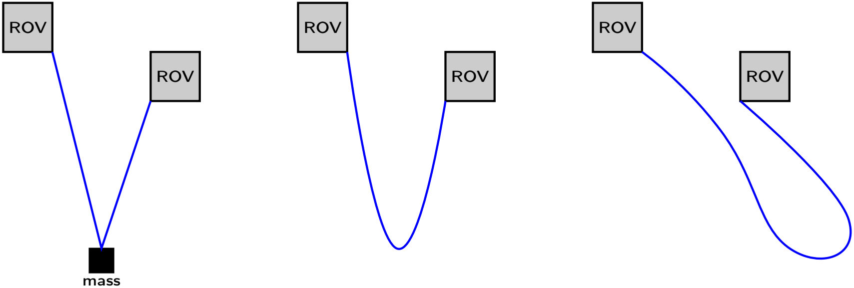


[1] Remotely Operated Vehicle

Underwater robot chain



State of the art



Constrained model Static model Dynamic model

Model, residual and parameter estimation

Catenary definition

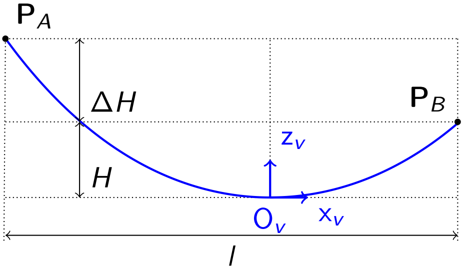


Figure: Catenary of length L hanging between $\mathbf{P}_A$ and $\mathbf{P}_B$.

$$\boxed{Z = \frac{\cosh\left(XC\right) - 1}{C}}$$

Catenary definition

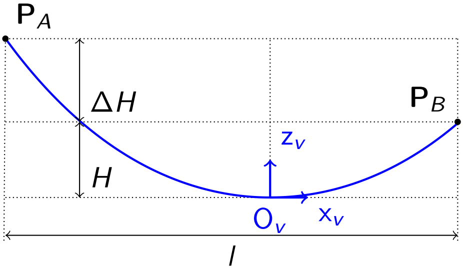


Figure: Catenary of length L hanging between $\mathbf{P}_A$ and $\mathbf{P}_B$.

$$Z = \frac{\cosh(XC) - 1}{C}$$

- Defined in a plane $(\mathbf{O}_v, \mathbf{x}_v, \mathbf{z}_v)$;

Catenary definition

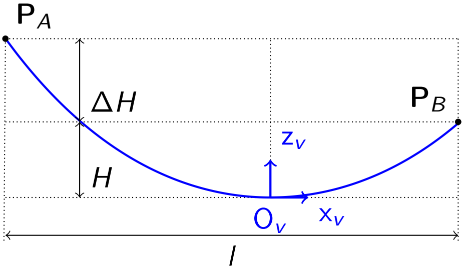


Figure: Catenary of length L hanging between $\mathbf{P}_A$ and $\mathbf{P}_B$.

$$\boxed{Z = \frac{\cosh(XC) - 1}{C}}$$

- Defined in a plane $(\mathbf{O}_v, \mathbf{x}_v, \mathbf{z}_v)$;
- only subjected to weight;

Catenary definition

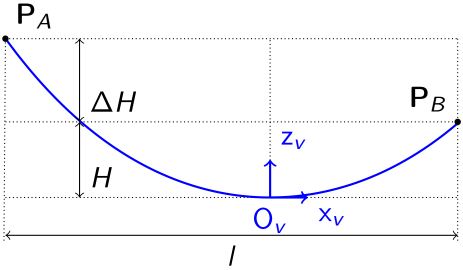


Figure: Catenary of length L hanging between $\mathbf{P}_A$ and $\mathbf{P}_B$.

$$\boxed{Z = \frac{\cosh(XC) - 1}{C}}$$

- Defined in a plane $(\mathbf{O}_v, \mathbf{x}_v, \mathbf{z}_v)$;
- only subjected to weight;
- homogeneous;

Catenary definition

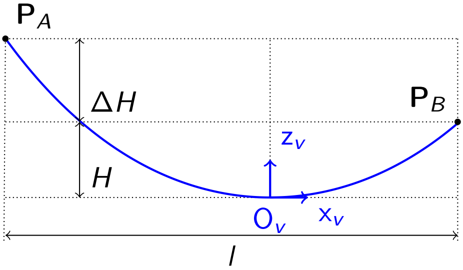


Figure: Catenary of length L hanging between $\mathbf{P}_A$ and $\mathbf{P}_B$.

$$\boxed{Z = \frac{\cosh(XC) - 1}{C}}$$

- Defined in a plane $(\mathbf{O}_v, \mathbf{x}_v, \mathbf{z}_v)$;
- only subjected to weight;
- homogeneous;
- no elasticity;

Catenary definition

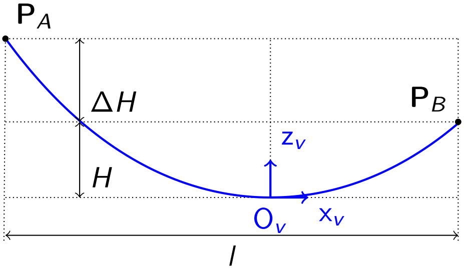


Figure: Catenary of length L hanging between $\mathbf{P}_A$ and $\mathbf{P}_B$.

$$\boxed{Z = \frac{\cosh\left(XC\right) - 1}{C}}$$

- Defined in a plane $(\mathbf{O}_v, \mathbf{x}_v, \mathbf{z}_v)$;
- only subjected to weight;
- homogeneous;
- no elasticity;
- no stiffness.

Degrees of freedom

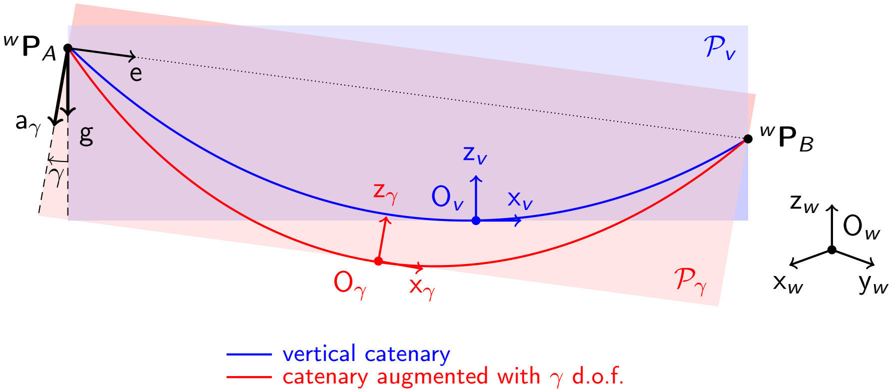


Figure: Standard and γ-augmented catenary [Drupt et al., 2022] of length L hanging between ${}^w\mathbf{P}_A$ and ${}^w\mathbf{P}_B$ in their respective planes.

Degrees of freedom

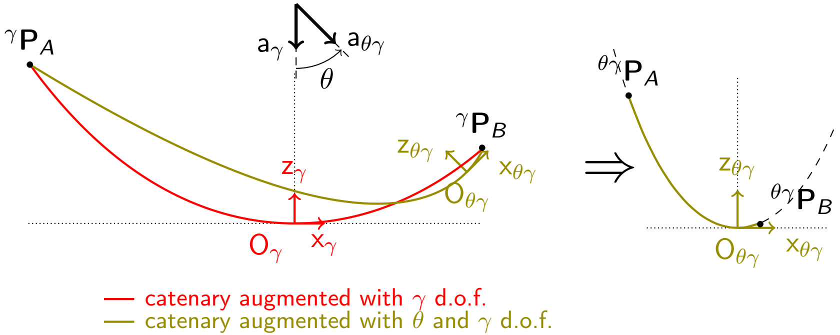


Figure: γ and $\theta\gamma$-augmented catenary of length L hanging between ${}^{w}\mathbf{P}_A$ and ${}^{w}\mathbf{P}_B$ in $\mathcal{P}_\gamma$.

Curvilinear discretization

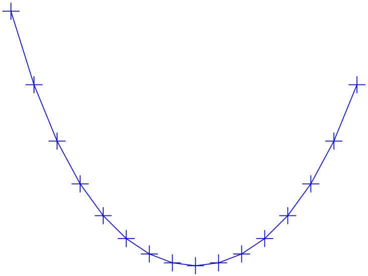


Figure: Normal discretization.

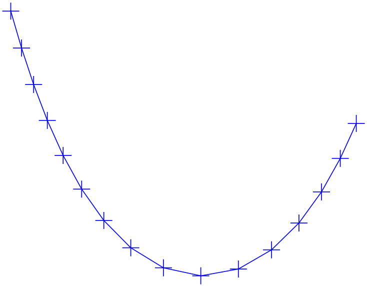


Figure: Curvilinear discretization.

$$^{w}\mathbf{P}_k^* = ({}^{w}X_k^*, {}^{w}Y_k^*, {}^{w}Z_k^*) \text{ with } * \in (m, v, \gamma, \theta\gamma) \text{ and } k \in \{0, \dots, n\}$$

Curvilinear discretization

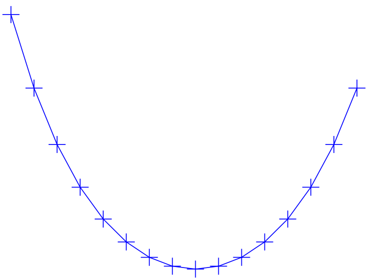


Figure: Normal discretization.

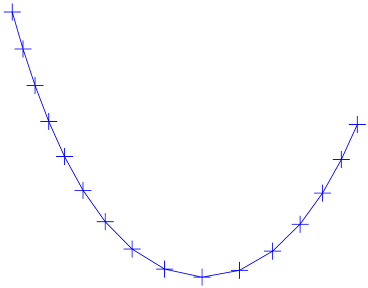


Figure: Curvilinear discretization.

$$ {}^{w}\mathbf{P}_k^* = ({}^{w}X_k^*, {}^{w}Y_k^*, {}^{w}Z_k^*) \text{ with } * \in \underbrace{(m, v, \gamma, \theta\gamma)}_{\text{measured}} \text{ and } k \in \{0, \dots, n\} $$

Curvilinear discretization

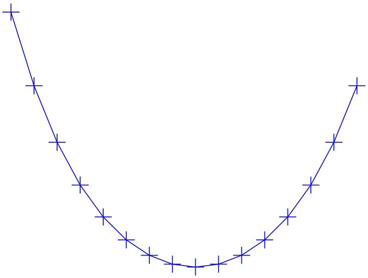


Figure: Normal discretization.

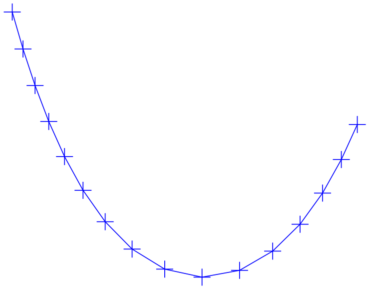


Figure: Curvilinear discretization.

$$ {}^{w}\mathbf{P}_k^* = ({}^{w}X_k^*, {}^{w}Y_k^*, {}^{w}Z_k^*) \text{ with } * \in (\underbrace{m, v}_{\text{vertical}}, \gamma, \theta\gamma) \text{ and } k \in \{0, \ldots, n\} $$

Curvilinear discretization

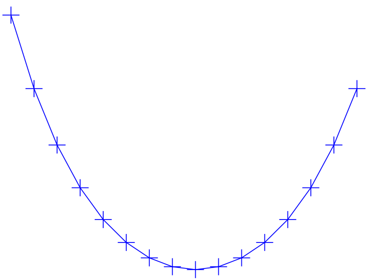


Figure: Normal discretization.

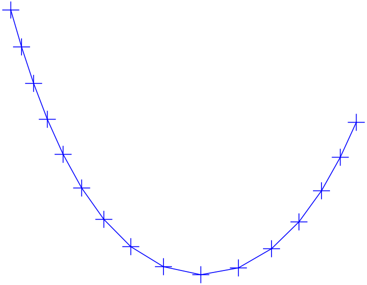


Figure: Curvilinear discretization.

$$^{w}\mathbf{P}_k^* = ({}^{w}X_k^*, {}^{w}Y_k^*, {}^{w}Z_k^*) \text{ with } * \in (\overbrace{m, v, \gamma, \theta\gamma}^{\gamma\text{-augmented}}) \text{ and } k \in \{0, \dots, n\}$$

Curvilinear discretization

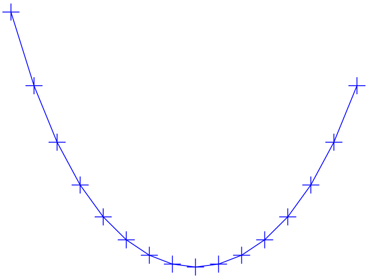


Figure: Normal discretization.

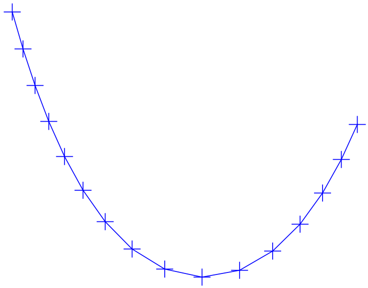


Figure: Curvilinear discretization.

$$^{w}\mathbf{P}_k^* = ({}^{w}X_k^*, {}^{w}Y_k^*, {}^{w}Z_k^*) \text{ with } * \in (m, v, \gamma, \overbrace{\theta\gamma}^{\theta\gamma\text{-augmented}}) \text{ and } k \in \{0, \ldots, n\}$$

Model accuracy

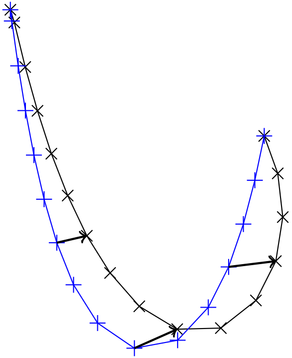


Figure: ${}^{w}\mathbf{P}_i^m$ and ${}^{w}\mathbf{P}_i^v$.

$$\varepsilon_{\mathbf{P}}^{v} = \frac{1}{n}\sum_{i=0}^{n} \left\| {}^{w}\mathbf{P}_i^m - {}^{w}\mathbf{P}_i^v \right\|$$

$$\varepsilon_{\mathbf{P}}^{\gamma} = \frac{1}{n}\sum_{i=0}^{n} \left\| {}^{w}\mathbf{P}_i^m - {}^{w}\mathbf{P}_i^\gamma \right\|$$

$$\varepsilon_{\mathbf{P}}^{\theta} = \frac{1}{n}\sum_{i=0}^{n} \left\| {}^{w}\mathbf{P}_i^m - {}^{w}\mathbf{P}_i^{\theta\gamma}\big|_{\gamma=0} \right\|$$

$$\varepsilon_{\mathbf{P}}^{\theta\gamma} = \frac{1}{n}\sum_{i=0}^{n} \left\| {}^{w}\mathbf{P}_i^m - {}^{w}\mathbf{P}_i^{\theta\gamma} \right\|$$

Parameters estimation

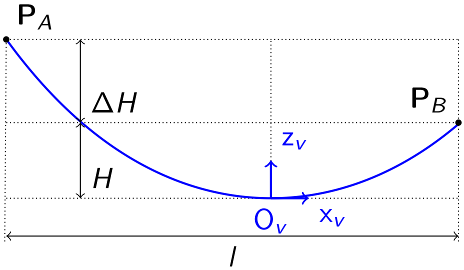


Figure: Catenary of length L hanging between $\mathbf{P}_A$ and $\mathbf{P}_B$.

$$C = \underset{C \in \mathbb{R}_+^*}{\operatorname{argmin}} \, C^2 \left(L^2 - \Delta H^2\right) - 4 \left(\cosh^2\left(\frac{lC}{2}\right) - 1\right)$$

Parameters estimation

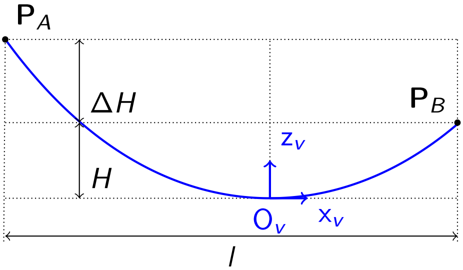


Figure: Catenary of length L hanging between $\mathbf{P}_A$ and $\mathbf{P}_B$.

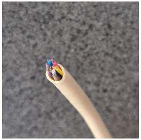
$$C = \underset{C \in \mathbb{R}_+^*}{\operatorname{argmin}} \, C^2 \left(L^2 - \Delta H^2\right) - 4 \left(\cosh^2\left(\frac{lC}{2}\right) - 1\right)$$

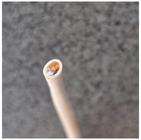
$$(\gamma, \theta) = \underset{(\gamma,\theta) \in [-\pi,\pi]^2}{\operatorname{argmin}} \, \varepsilon_{\mathbf{P}}^{\theta\gamma}(\gamma, \theta)$$

Experiments

Candidate cables


(1)


(2)


(3)


(4)


(5)


(6)


(7)


(8)

Figure: Pictures of the different cables used in the experiments: (1) coaxial cable; (2) four pairs ethernet cable; (3) two pairs ethernet cable; (4) floating rope; (5) rope; (6) weighted rope; (7) steel chain; (8) elastic rope.

Experimental setup

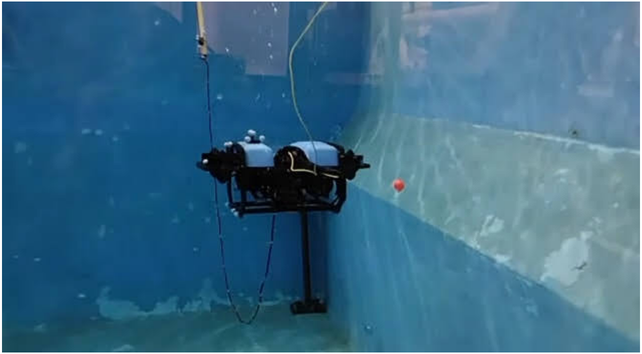


Figure: Picture of the robot and the cable while doing experiments.

The whole system is tracked at 100 Hz with a five cameras Qualisys motion capture system.
Visual markers are on the robot and the cables spaced out by 20 cm.

Description of the experiments

Three experimental parameters:

1. direction of motion: surge or sway;

Description of the experiments

Three experimental parameters:

1. direction of motion: surge or sway;
2. speed of the robot: $0.6\,\mathrm{m\,s^{-1}}$ or $0.3\,\mathrm{m\,s^{-1}}$;

Description of the experiments

Three experimental parameters:

1. direction of motion: surge or sway;
2. speed of the robot: $0.6\,\mathrm{m\,s^{-1}}$ or $0.3\,\mathrm{m\,s^{-1}}$;
3. initial distance between attachment points: $1.5\,\mathrm{m}$ or $2.0\,\mathrm{m}$.

Description of the experiments

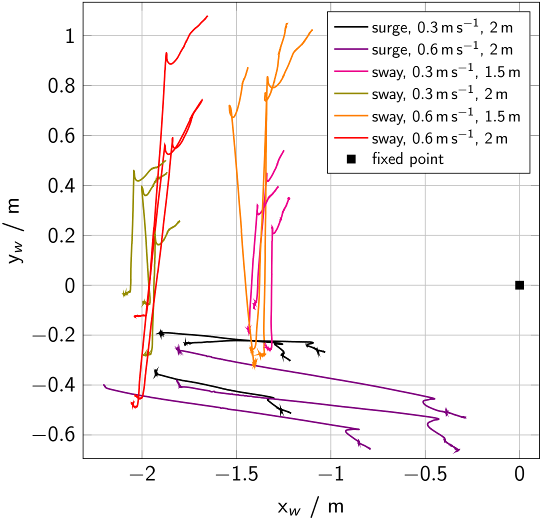


Figure: View of the trajectories for cable 6.

General results

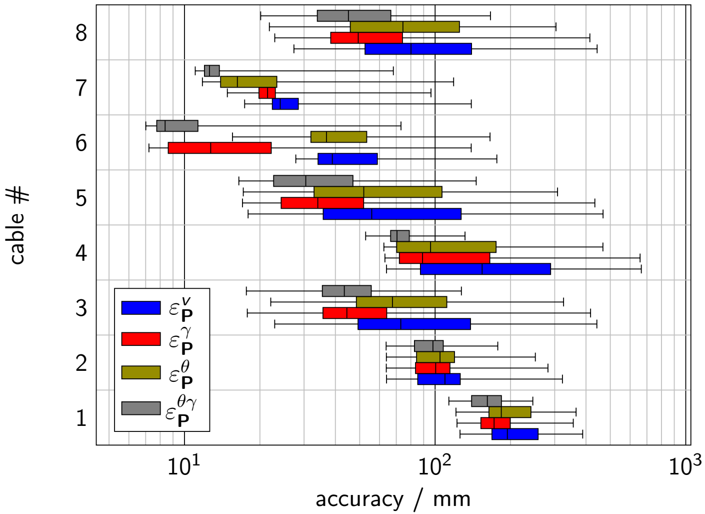


Figure: Accuracy of the models for each cable.

Cable specific results

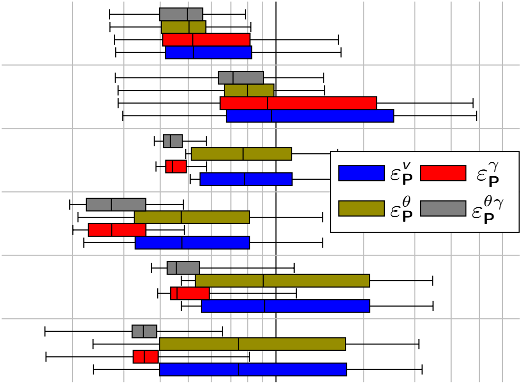
Experimental parameters	Box plot
surge, $0.3\,\mathrm{m\,s^{-1}}$, $2.0\,\mathrm{m}$	
surge, $0.6\,\mathrm{m\,s^{-1}}$, $2.0\,\mathrm{m}$	
sway, $0.3\,\mathrm{m\,s^{-1}}$, $1.5\,\mathrm{m}$	
sway, $0.3\,\mathrm{m\,s^{-1}}$, $2.0\,\mathrm{m}$	
sway, $0.6\,\mathrm{m\,s^{-1}}$, $1.5\,\mathrm{m}$	
sway, $0.6\,\mathrm{m\,s^{-1}}$, $2.0\,\mathrm{m}$	
accuracy / mm	10^2

Table: Accuracy of the models for cable 3.

Cable specific results

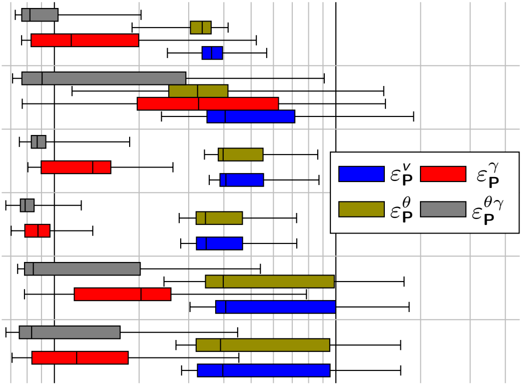
Experimental parameters	Box plot
surge, $0.3\,\mathrm{m\,s^{-1}}$, $2.0\,\mathrm{m}$	
surge, $0.6\,\mathrm{m\,s^{-1}}$, $2.0\,\mathrm{m}$	
sway, $0.3\,\mathrm{m\,s^{-1}}$, $1.5\,\mathrm{m}$	
sway, $0.3\,\mathrm{m\,s^{-1}}$, $2.0\,\mathrm{m}$	
sway, $0.6\,\mathrm{m\,s^{-1}}$, $1.5\,\mathrm{m}$	
sway, $0.6\,\mathrm{m\,s^{-1}}$, $2.0\,\mathrm{m}$	
accuracy / mm	10^1 10^2

Table: Accuracy of the models for cable 6.

Cable specific results

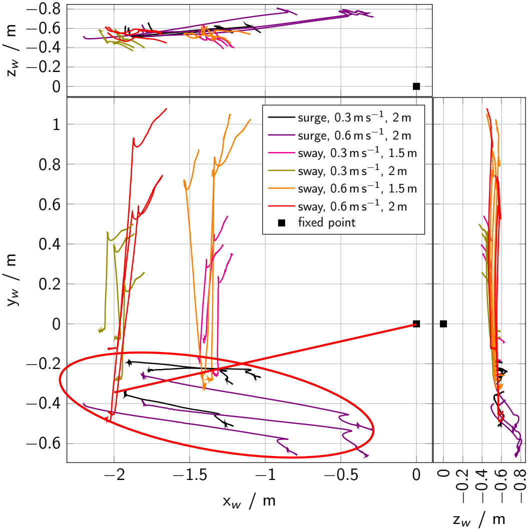


Figure: View of the trajectories for cable 6.

Cable specific results

Experimental parameters	Box plot
surge, $0.3\,\mathrm{m\,s^{-1}}$, $2.0\,\mathrm{m}$	
surge, $0.6\,\mathrm{m\,s^{-1}}$, $2.0\,\mathrm{m}$	
sway, $0.3\,\mathrm{m\,s^{-1}}$, $1.5\,\mathrm{m}$	
sway, $0.3\,\mathrm{m\,s^{-1}}$, $2.0\,\mathrm{m}$	
sway, $0.6\,\mathrm{m\,s^{-1}}$, $1.5\,\mathrm{m}$	
sway, $0.6\,\mathrm{m\,s^{-1}}$, $2.0\,\mathrm{m}$	
accuracy / mm	10^1 10^2

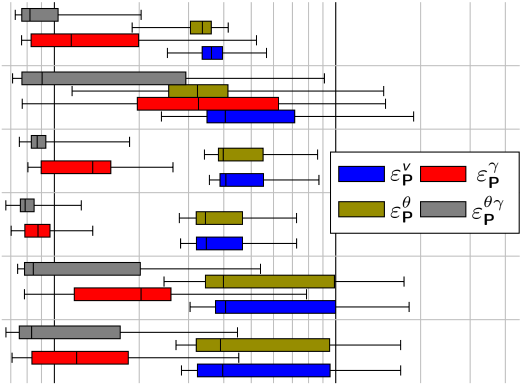


Table: Accuracy of the models for cable 6.

Single sequence results

Video!

Conclusion

- The augmented catenary provides a better estimation for all cables compared to the standard catenary;

Conclusion

- The augmented catenary provides a better estimation for all cables compared to the standard catenary;
- Catenary models are most relevant for:
  - Cables that are flexible (no plastic deformations);

Conclusion

- The augmented catenary provides a better estimation for all cables compared to the standard catenary;
- Catenary models are most relevant for:
  - Cables that are flexible (no plastic deformations);
  - Cables that are adequately heavy (cable 6 weights $0.39\,\mathrm{N\,m^{-1}}$ in water);

Conclusion

- The augmented catenary provides a better estimation for all cables compared to the standard catenary;
- Catenary models are most relevant for:
    - Cables that are flexible (no plastic deformations);
    - Cables that are adequately heavy (cable 6 weights $0.39\,\mathrm{N\,m^{-1}}$ in water);
    - Slow motion and low dynamics (for best accuracy).

Conclusion

- The augmented catenary provides a better estimation for all cables compared to the standard catenary;
- Catenary models are most relevant for:
  - Cables that are flexible (no plastic deformations);
  - Cables that are adequately heavy (cable 6 weights $0.39\,\mathrm{N\,m^{-1}}$ in water);
  - Slow motion and low dynamics (for best accuracy).
- Cables that fit the catenary model have the most gains from our augmentations;

Conclusion

- The augmented catenary provides a better estimation for all cables compared to the standard catenary;
- Catenary models are most relevant for:
  - Cables that are flexible (no plastic deformations);
  - Cables that are adequately heavy (cable 6 weights $0.39\,\mathrm{N\,m^{-1}}$ in water);
  - Slow motion and low dynamics (for best accuracy).
- Cables that fit the catenary model have the most gains from our augmentations;
- The accuracy of the augmented catenary model for duly chosen cables can be as low as a centimeter (median, cable 6).

Conclusion

- The augmented catenary provides a better estimation for all cables compared to the standard catenary;
- Catenary models are most relevant for:
  - Cables that are flexible (no plastic deformations);
  - Cables that are adequately heavy (cable 6 weights $0.39\,\mathrm{N\,m^{-1}}$ in water);
  - Slow motion and low dynamics (for best accuracy).
- Cables that fit the catenary model have the most gains from our augmentations;
- The accuracy of the augmented catenary model for duly chosen cables can be as low as a centimeter (median, cable 6).

Current work: proposing a way to describe the dynamics of the new degrees of freedom.

[Drupt et al., 2022] Drupt, J., Dune, C., Comport, A. I., and Hugel, V. (2022).
Validity of the catenary model for moving submarine cables with negative buoyancy.
In *3rd workshop on RObotic MAnipulation of Deformable Objects: challenges in perception, planning and control for Soft Interaction (ROMADO-SI)*, Kyoto, Japan.