Reliable SLAM in ambiguous environments

SWIM

30th June 2025, Rennes, France

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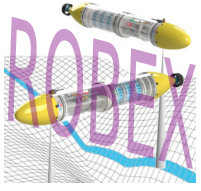
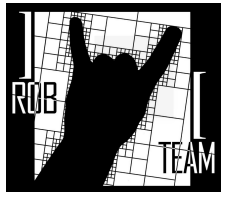
ENSTA, Lab-STICC, Brest, France

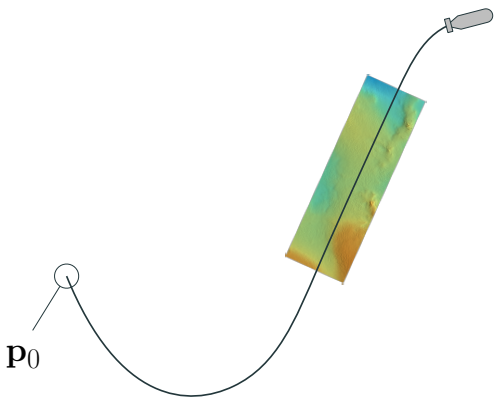
Joint work with Luc Jaulin, Peter Franek and Michel Legris

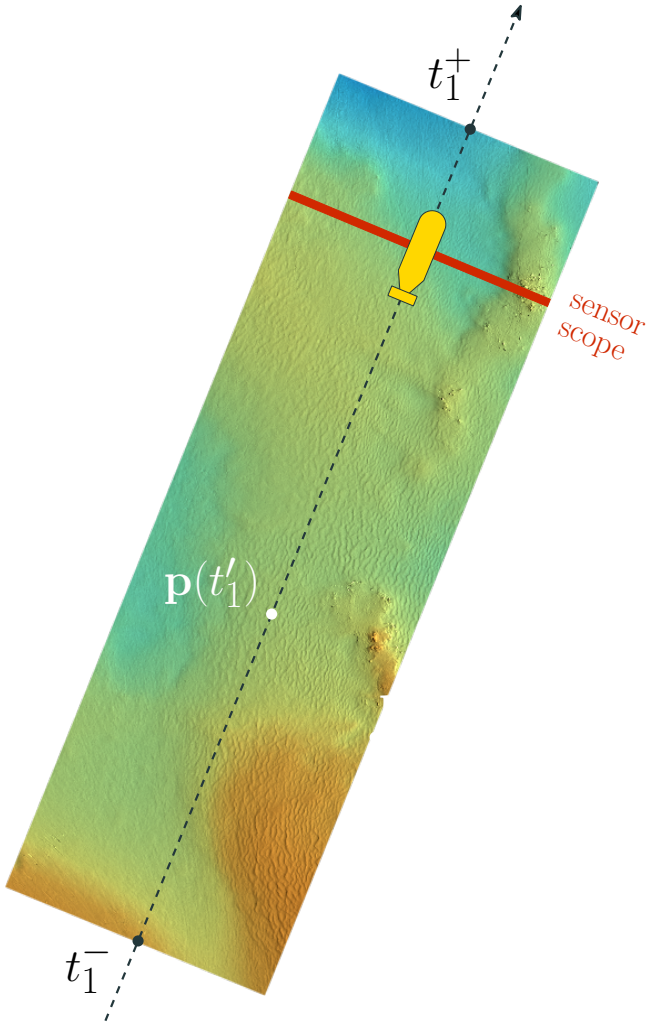




The SLAM problem

Loop closure with ground perception





Robot exploring an environment...

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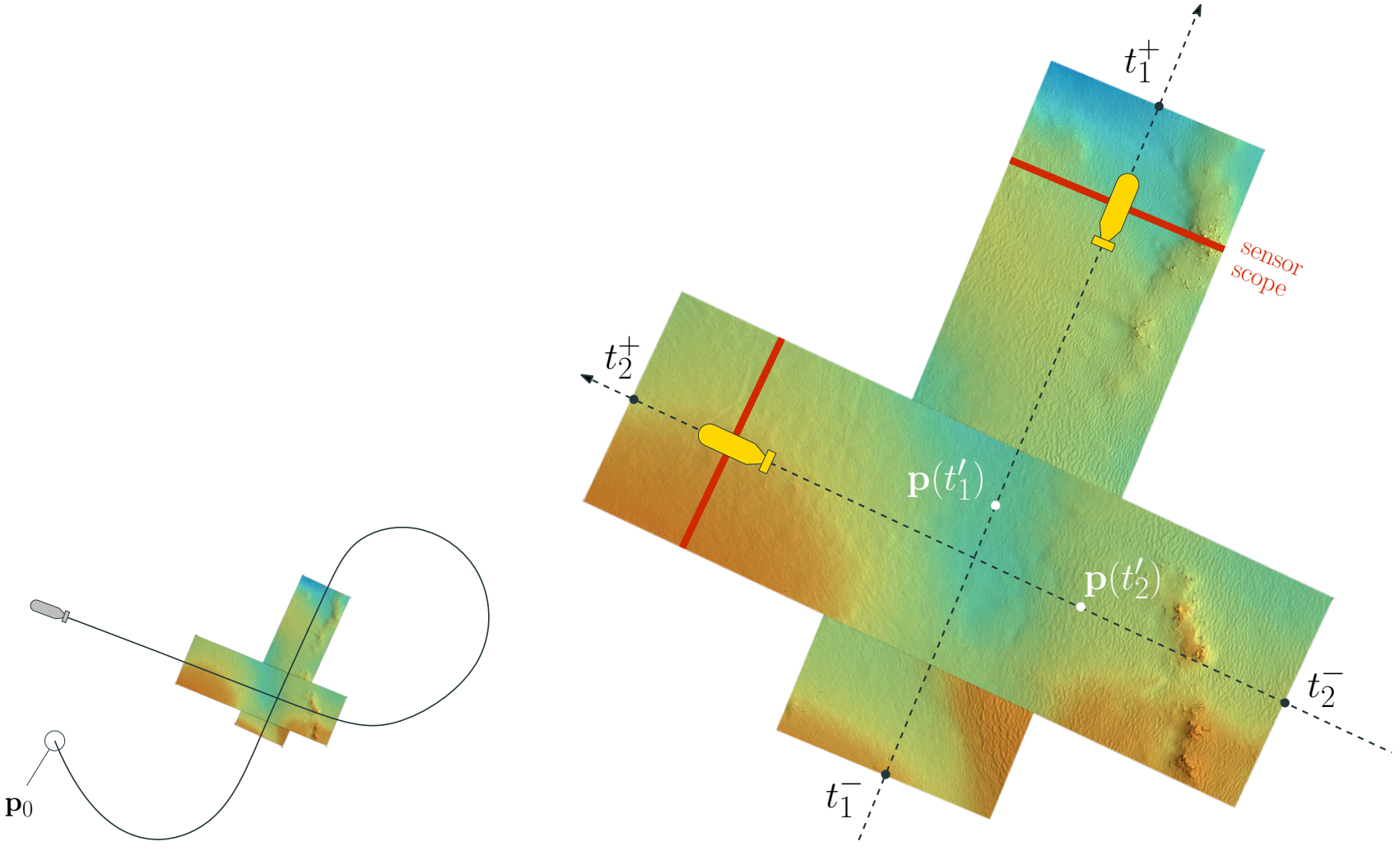

Robot exploring an environment... coming back to a previous position...

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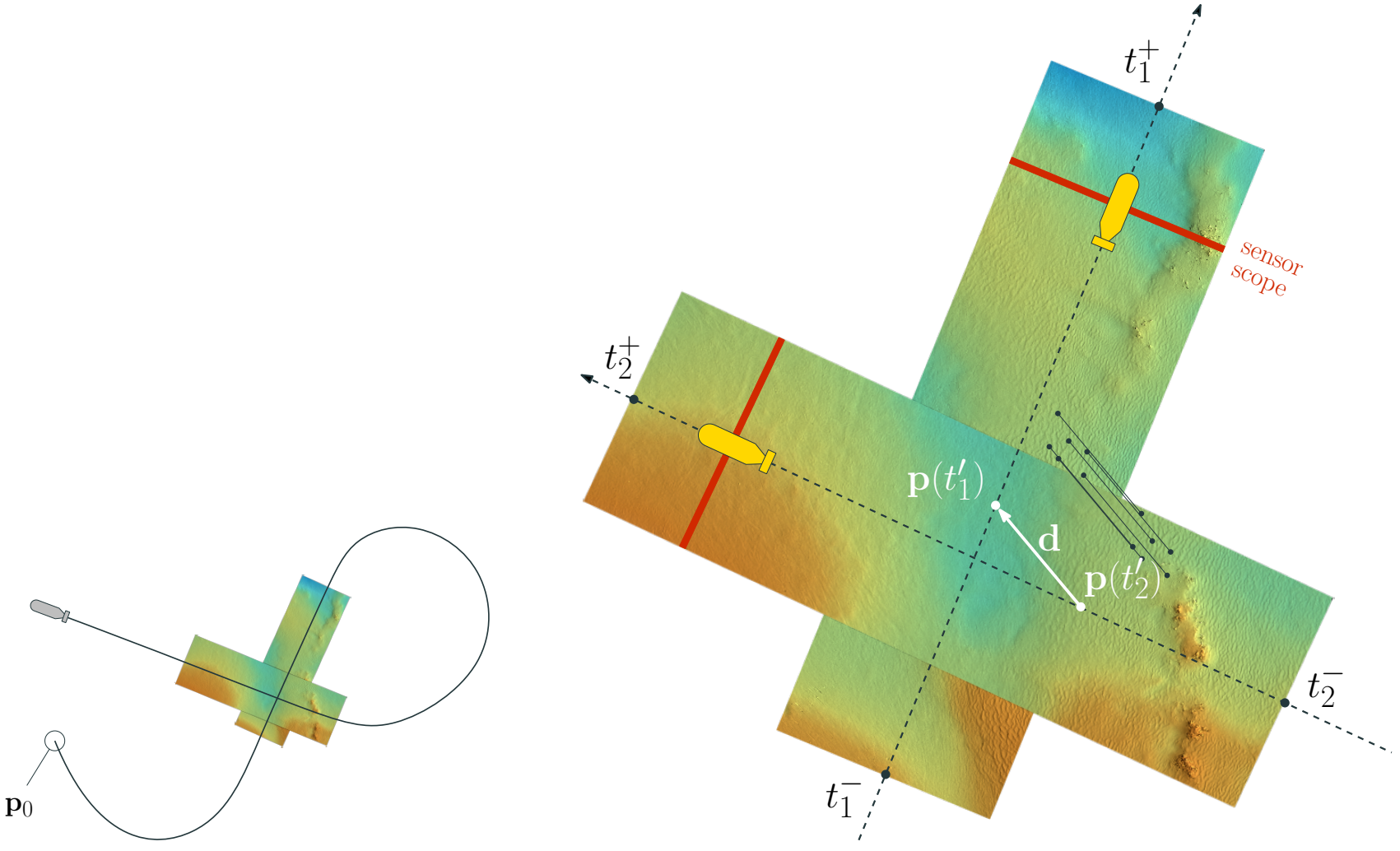
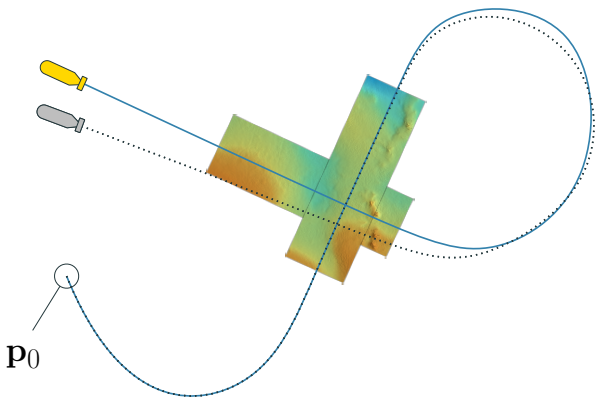
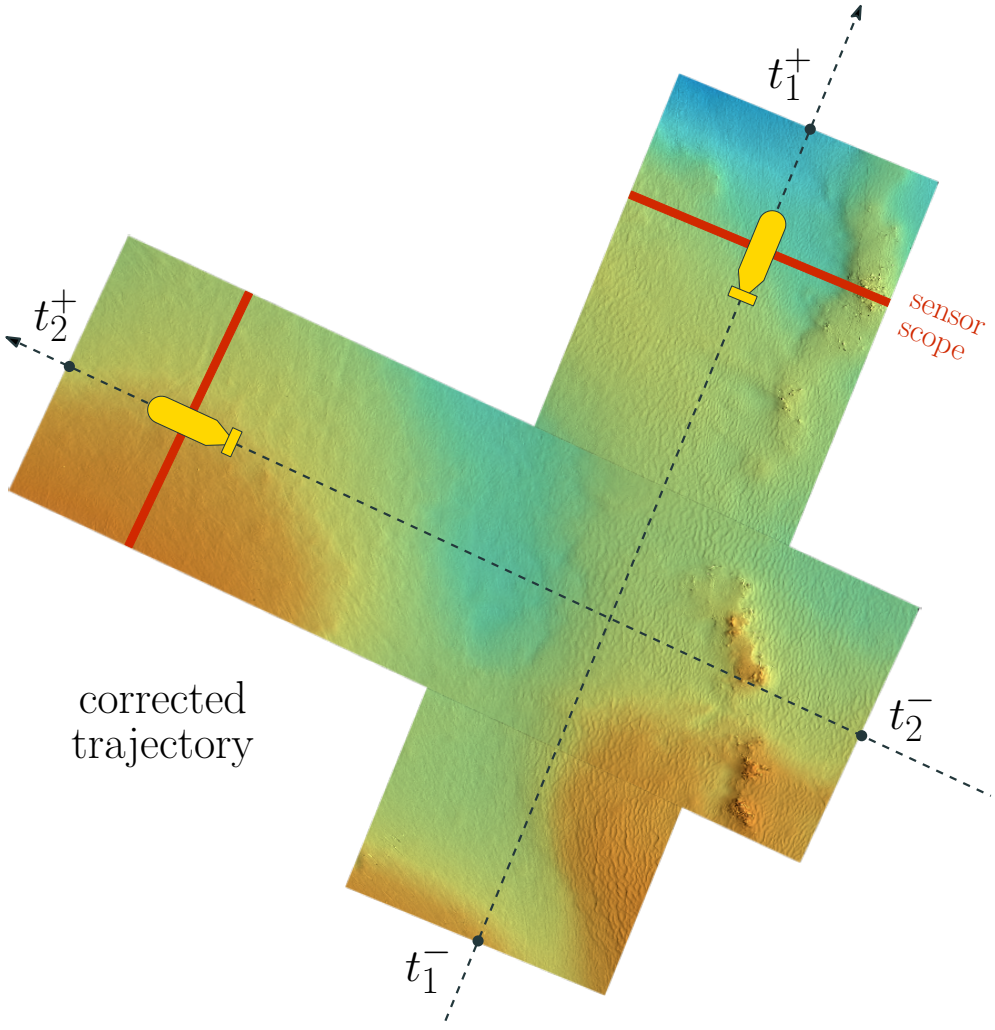


Robot exploring an environment... coming back to a previous position... detecting loop closure...

Loop closure with ground perception

SLAM:
Simultaneous
Localization
And Mapping



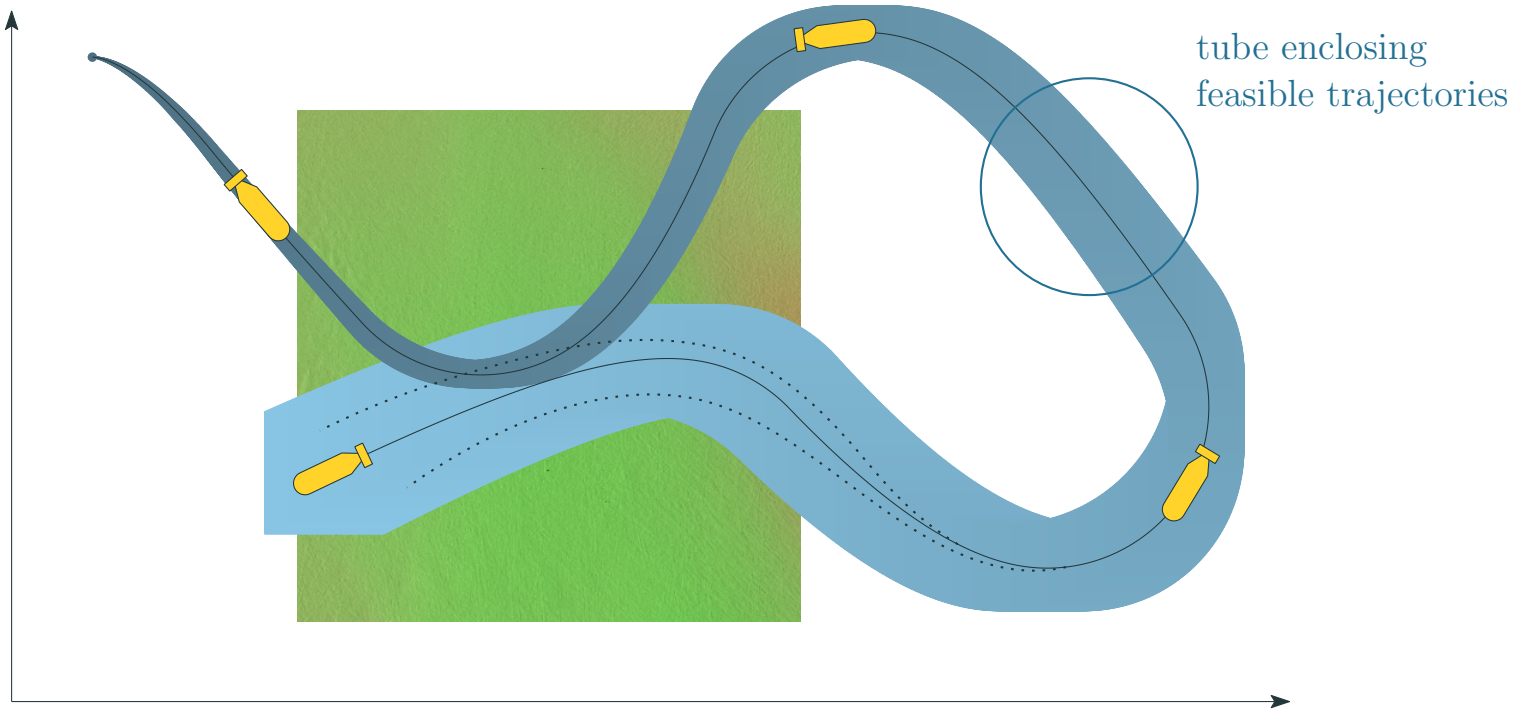


Robot exploring an environment... coming back to a previous position... detecting loop closure... correcting its trajectory.

Problem: doubtful loops and similar environments

What if we recognize a **wrong scene**?

- homogeneous environments $\Longrightarrow$ similar observations
- strong positioning drift $\Longrightarrow$ false loop detections



Doubtful loop: the actual trajectory did not loop, but the evolution uncertainties and the homogeneity of the environment may lead to an incorrect correction.

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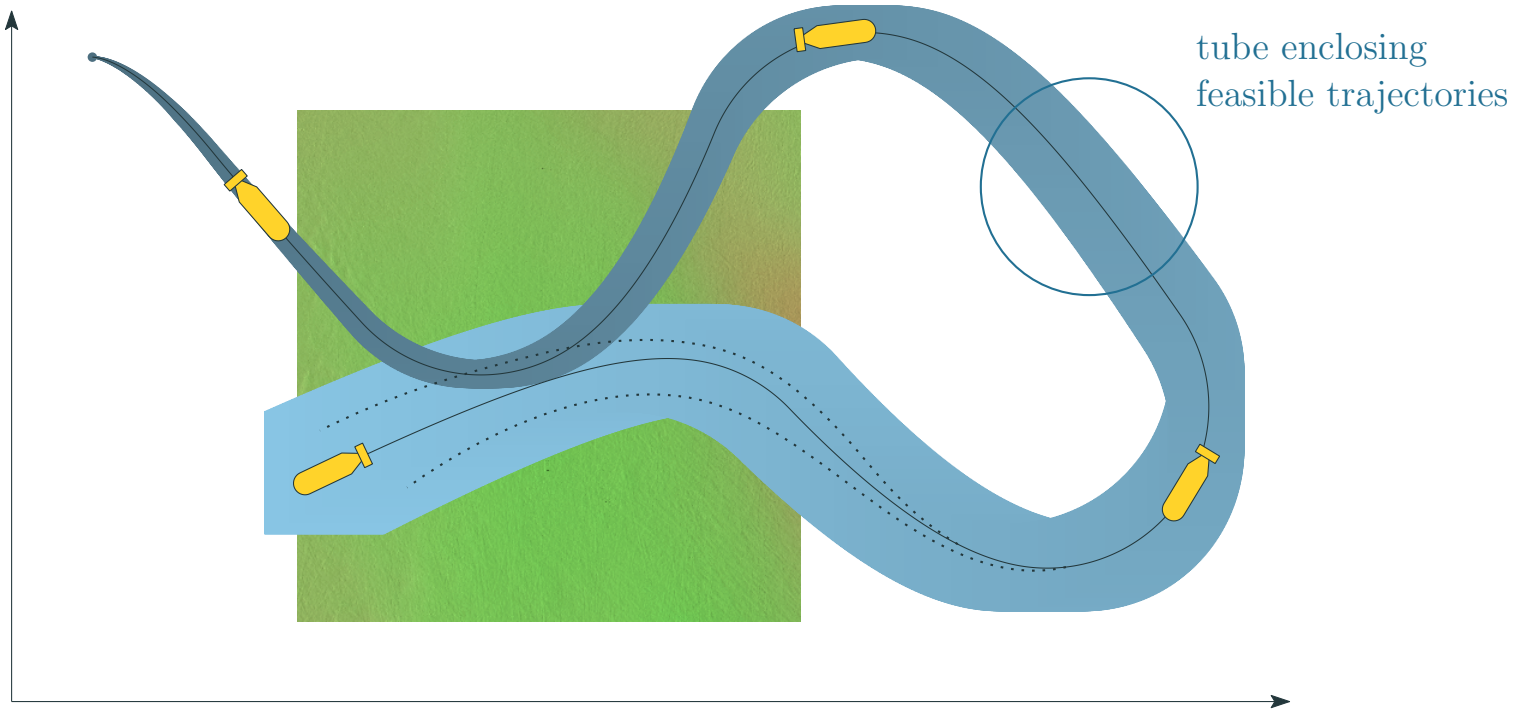
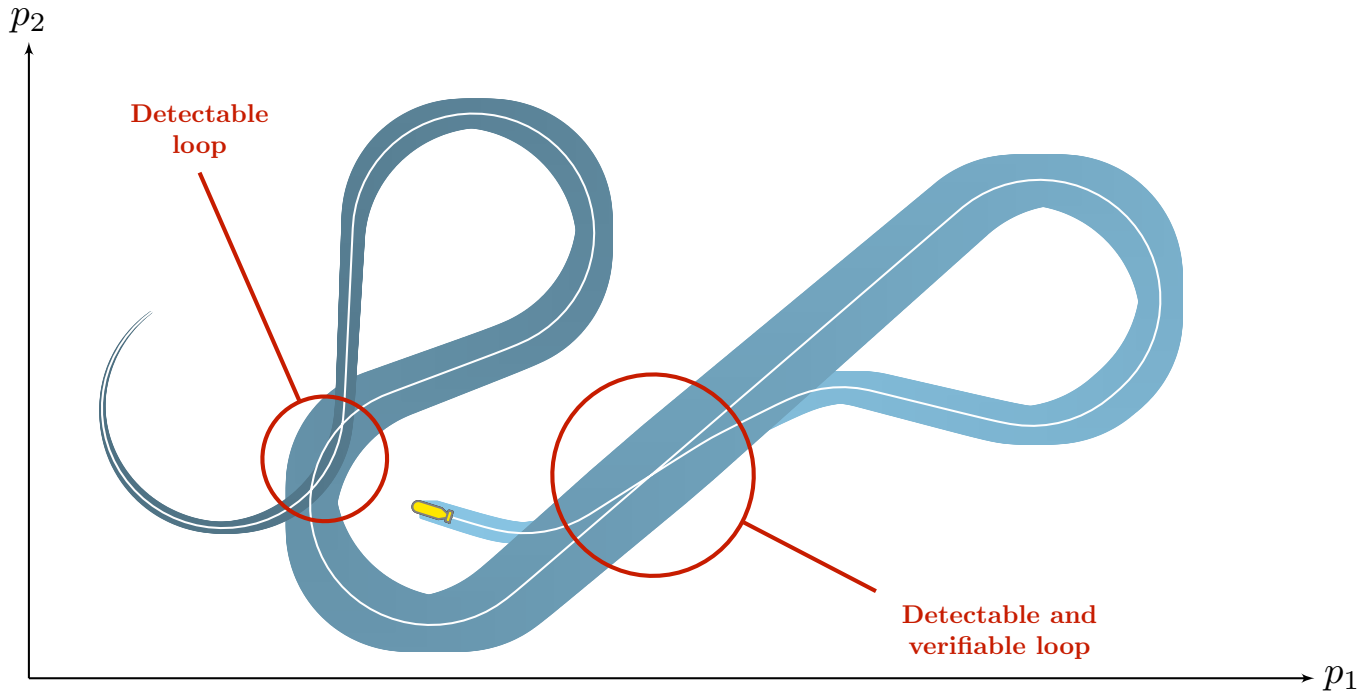


Doubtful loop: the actual trajectory did not loop, but the evolution uncertainties and the homogeneity of the environment may lead to an incorrect correction. $\Longrightarrow$ *significant impact on the next SLAM iterations.*

Uncertainties: detection vs verification

Uncertain trajectories are enclosed by **tubes**.



Only 1 loop can be verified – at least 2 feasible loops are detected.

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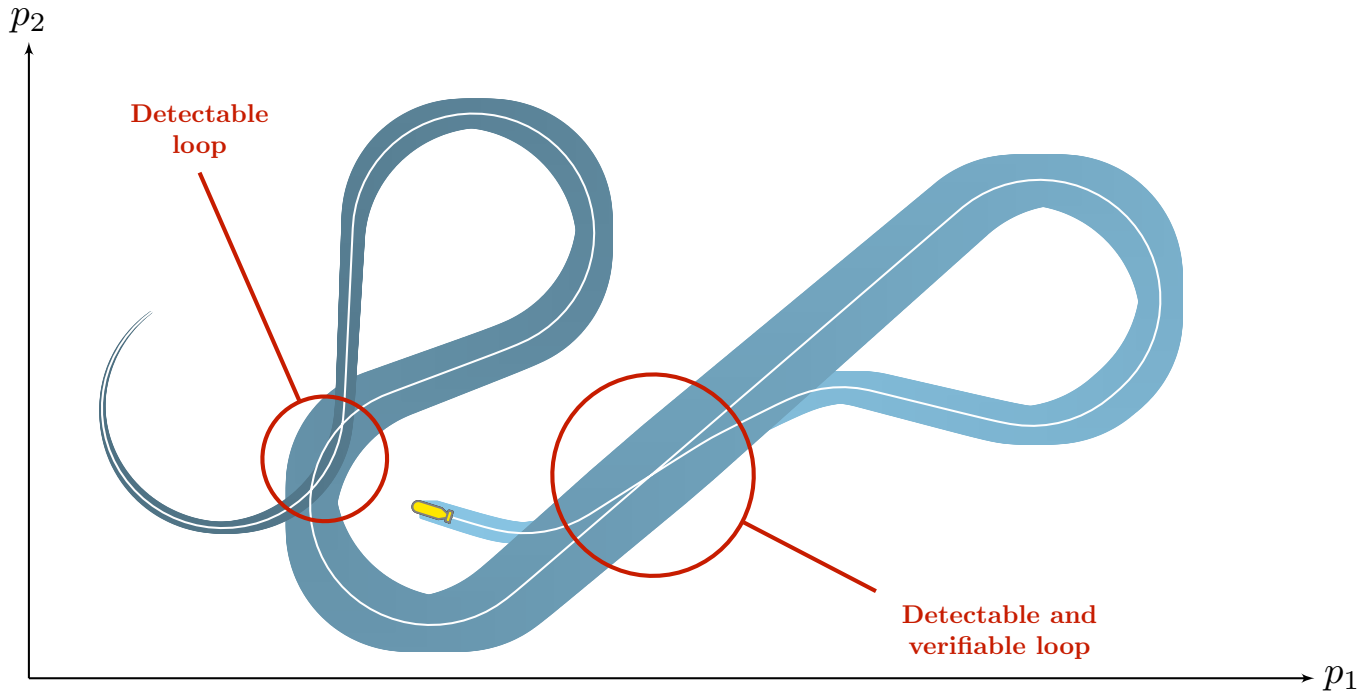

Only 1 loop can be verified – at least 2 feasible loops are detected.

Need for **loop proof**:

- verify that a trajectory crosses itself at some point
- ..whatever the uncertainties describing this trajectory

Set of feasible loops

Definitions (Aubry, 2013)

- robot position: $\mathbf{p} = (x, y)^\intercal \in \mathbb{R}^2$
- 2D robot trajectory: $\mathbf{p}(t) : \mathbb{R} \to \mathbb{R}^2,\ t \in [t_0, t_f]$
- looped trajectory $\Leftrightarrow$ trajectory that crosses itself
  - $\mathbf{p}(t_1) = \mathbf{p}(t_2),\ t_1 \neq t_2$
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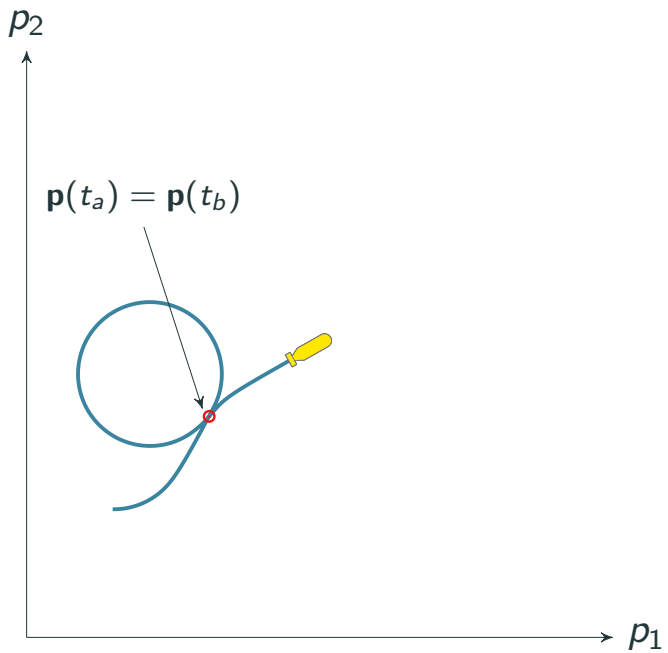

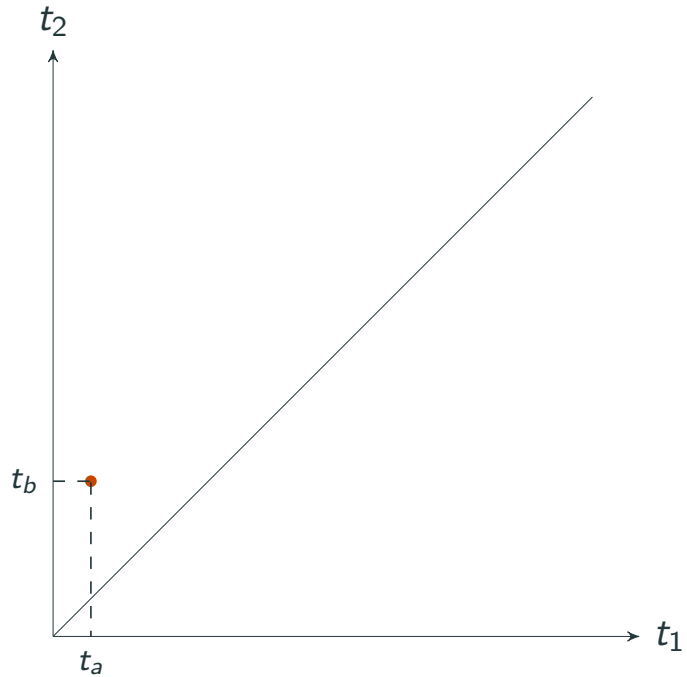

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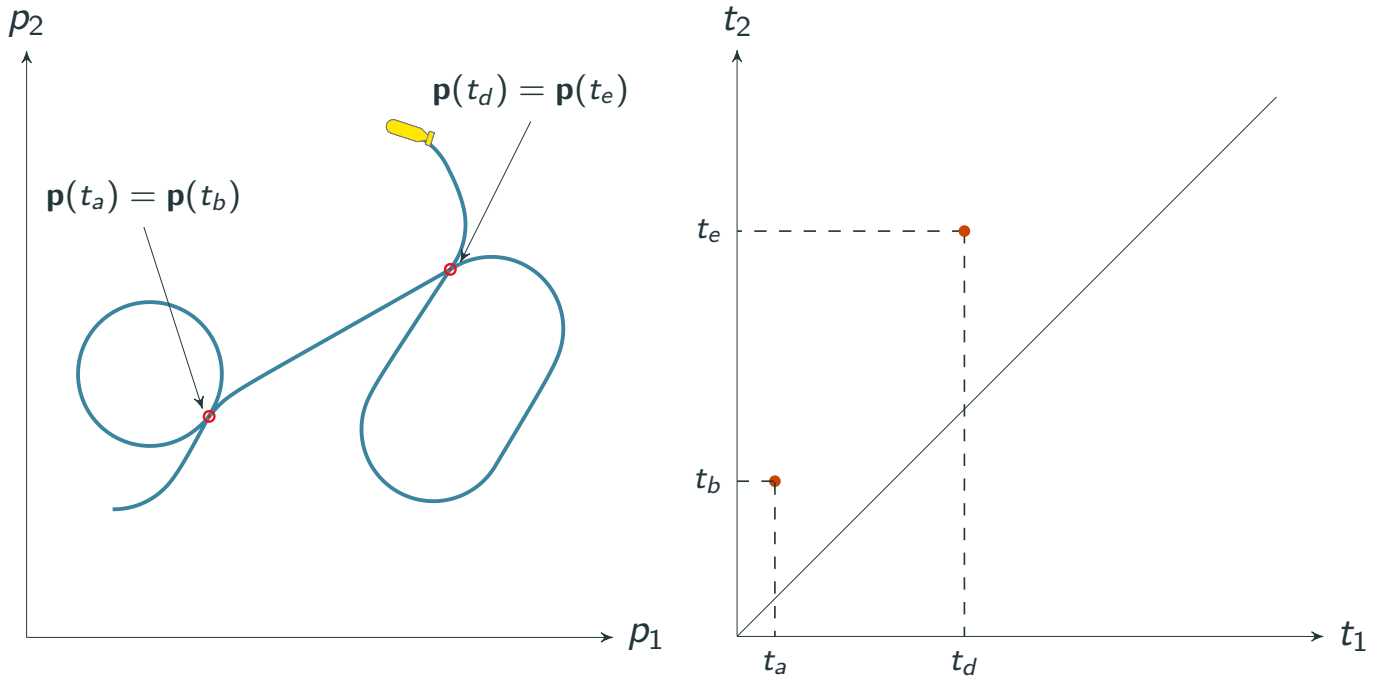

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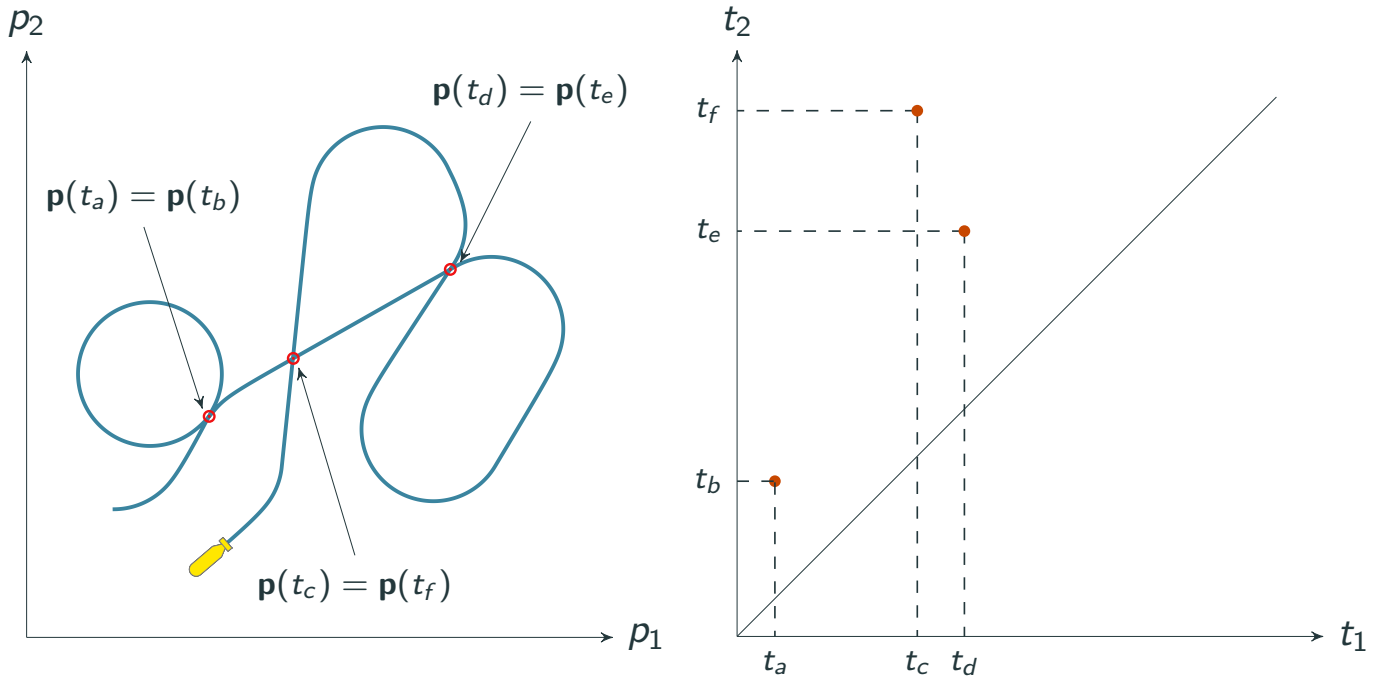
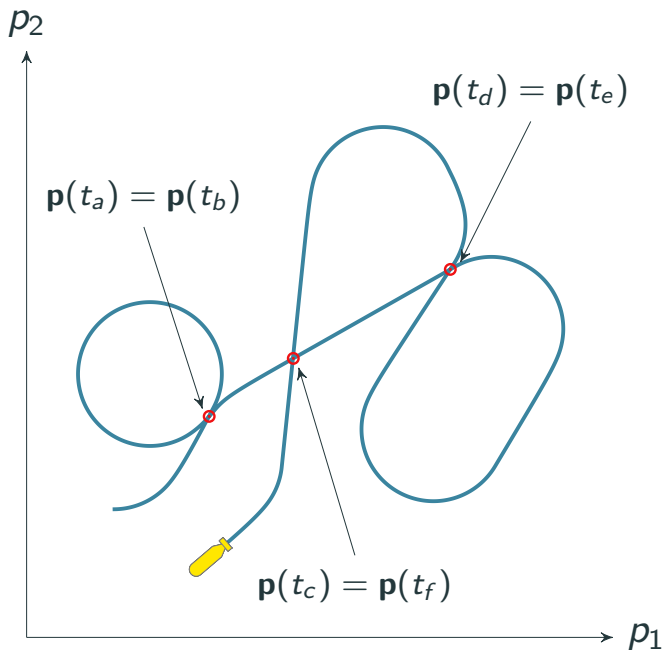
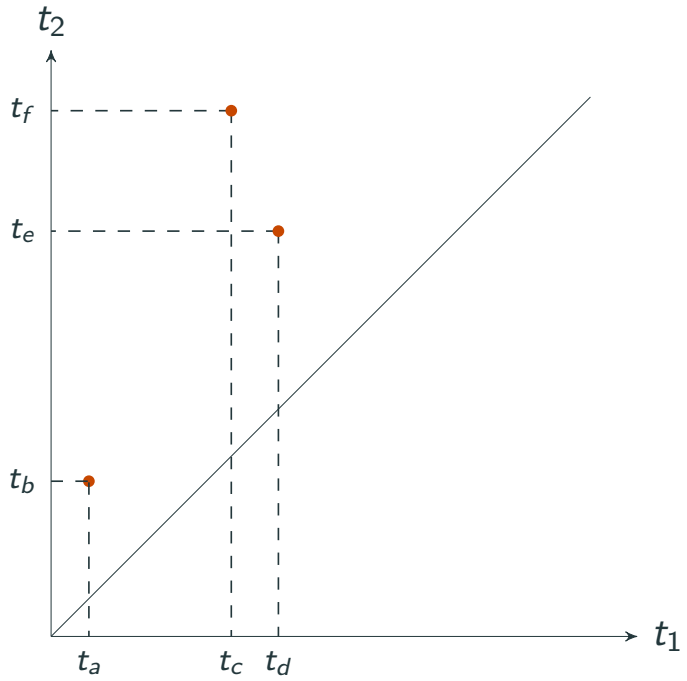


Definitions (Aubry, 2013)

- t-plane $\Leftrightarrow$ all feasible t-pairs $= [t_0, t_f]^2$
- loop set $\mathbb{T}^*$:
  - $\mathbb{T}^* = \left\{(t_1, t_2) \in [t_0, t_f]^2 \mid \mathbf{p}(t_1) = \mathbf{p}(t_2), t_1 < t_2\right\}$
- loop set of below example:
  - $\mathbb{T}^* = \{(t_a, t_b), (t_c, t_f), (t_d, t_e)\}$





Computing loops from robot sensors

Context: robot trajectory $\mathbf{p}(t)$ cannot be directly sensed.
Computation from speed measurements:

$$\mathbf{p}(t) = \int_{t_0}^{t} \mathbf{v}(\tau) d\tau + \mathbf{p}_0, \tag{1}$$

with $\mathbf{v}(t) \in \mathbb{R}^2$: robot velocity vector at time $t \in [t_0, t_f]$.

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Loop-set from velocity:

$$\begin{aligned} \mathbb{T}^* &= \left\{ (t_1, t_2) \in [t_0, t_f]^2 \mid \mathbf{p}^*(t_1) = \mathbf{p}^*(t_2), t_1 < t_2 \right\} && (2) \\ &= \left\{ (t_1, t_2) \in [t_0, t_f]^2 \mid \int_{t_1}^{t_2} \mathbf{v}^*(\tau) d\tau = \mathbf{0}, t_1 < t_2 \right\} && (3) \end{aligned}$$

Bounded-error context: $\mathbb{T}$

Actual loop-set $\mathbb{T}^*$ (error free):

$$\mathbb{T}^* = \left\{ (t_1, t_2) \mid \int_{t_1}^{t_2} \mathbf{v}^*(\tau) d\tau = \mathbf{0} \right\} \tag{4}$$

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Set-membership approach:

$$\mathbb{T}^* \subset \mathbb{T} \subset [t_0, t_f]^2 \tag{6}$$

Algorithm for detecting loops [Aubry 2013]

Using a SIVIA-like algorithm, we define the following inclusion tests:

$$\left.\begin{array}{r} [t_1]-[t_2]\subset\mathbb{R}^- \\ \int_{[t_1]}^{[t_2]}\mathbf{v}^-(\tau)d\tau \leqslant \mathbf{0} \leqslant \int_{[t_1]}^{[t_2]}\mathbf{v}^+(\tau)d\tau \end{array}\right\} \Rightarrow [\mathbf{t}]\subset\mathbb{T}$$

$$\left.\begin{array}{r} [t_1]-[t_2]\subset\mathbb{R}^+ \\ \mathbf{0}\notin \int_{[t_1]}^{[t_2]}[\mathbf{v}](\tau)d\tau \end{array}\right\} \Rightarrow [\mathbf{t}]\cap\mathbb{T}=\varnothing$$

Example of application

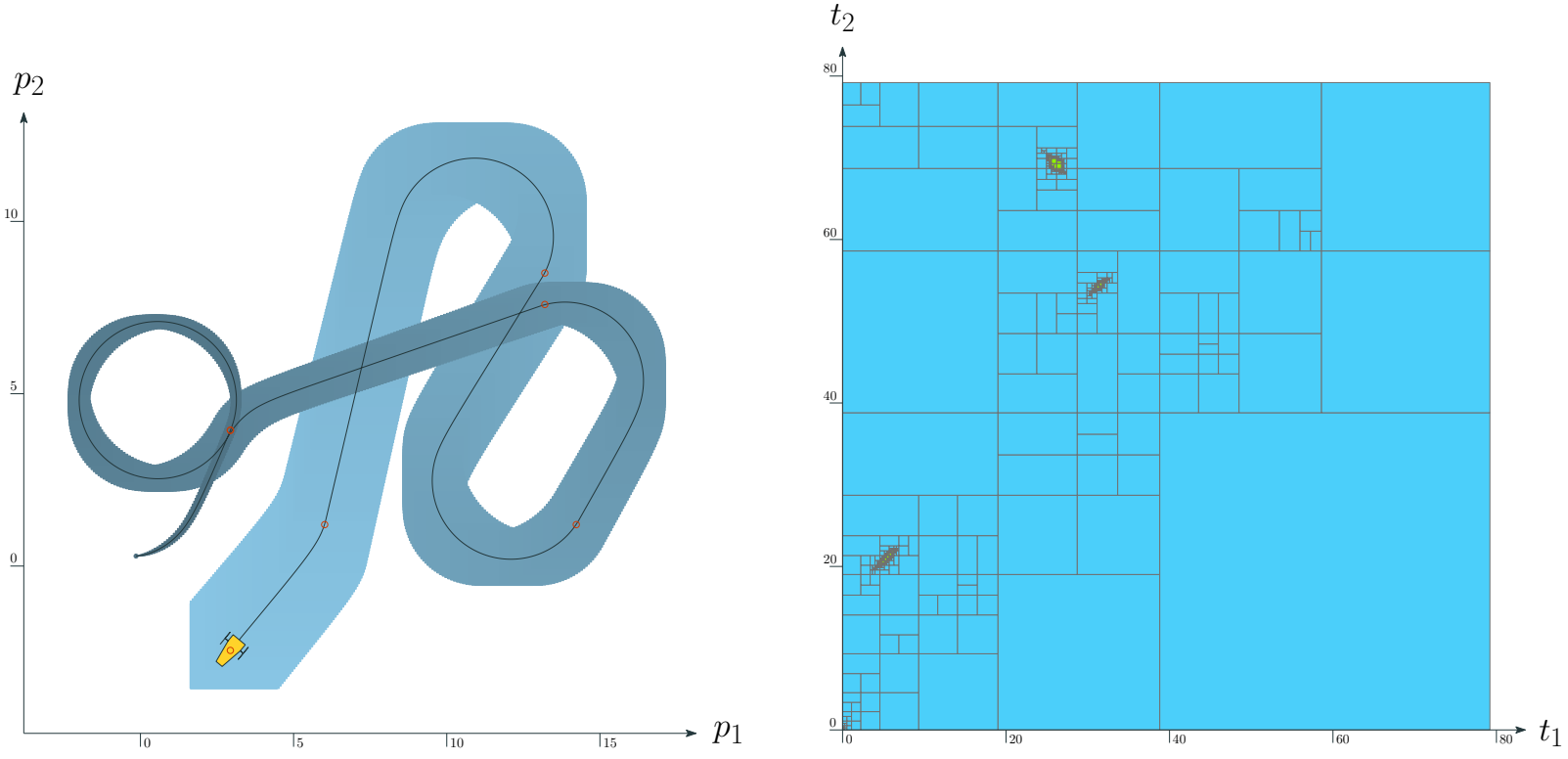
www.codac.io

Computing an enclosure $\mathbb{T}$ of the loop set using Codac:

```
BoolInterval test_loop(const IntervalVector& t)
{
    auto partial_integ = v.partial_integral(t[0],t[1]);

    if((t[0]-t[1]).is_strict_subset({0,oo})
          || !v.integral(t[0],t[1]).interior_contains({0,0})
          || !v(t[0]|t[1]).interior_contains({0,0}))
       return BoolInterval::FALSE;

    else if((t[0]-t[1]).is_strict_subset({-oo,0})
          && partial_integ.first.is_strict_subset({{-oo,0},{-oo,0}})
          && partial_integ.second.is_strict_subset({{0,oo},{0,oo}}))
       return BoolInterval::TRUE;

    else
       return BoolInterval::UNKNOWN;
}

IntervalVector T { {0,10}, {0,10} };
auto loops = regular_pave(T, test_loop, 1e-1);
```

Example of application

www.codac.io



Example of computed t-plane.

Example of application

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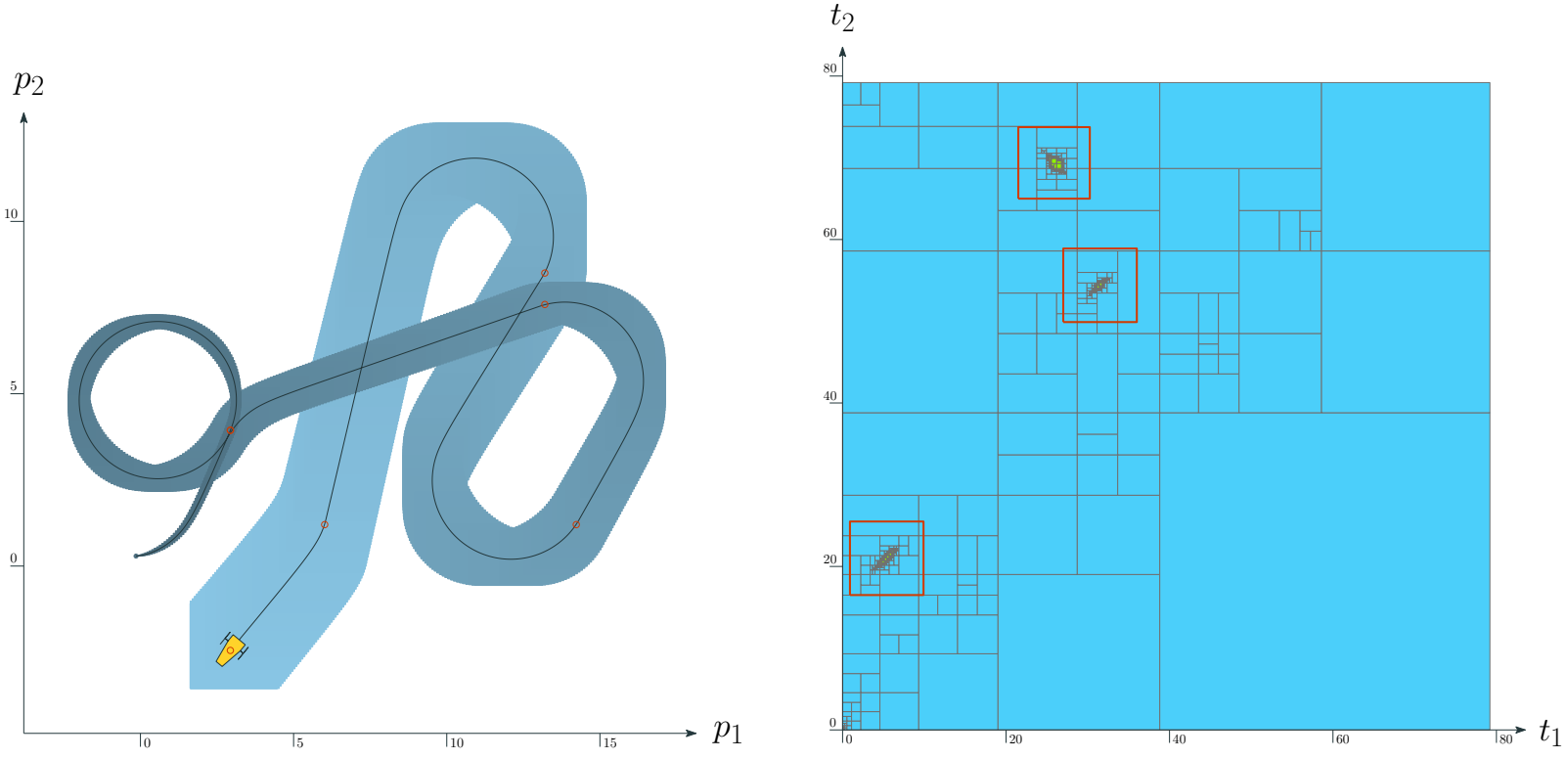

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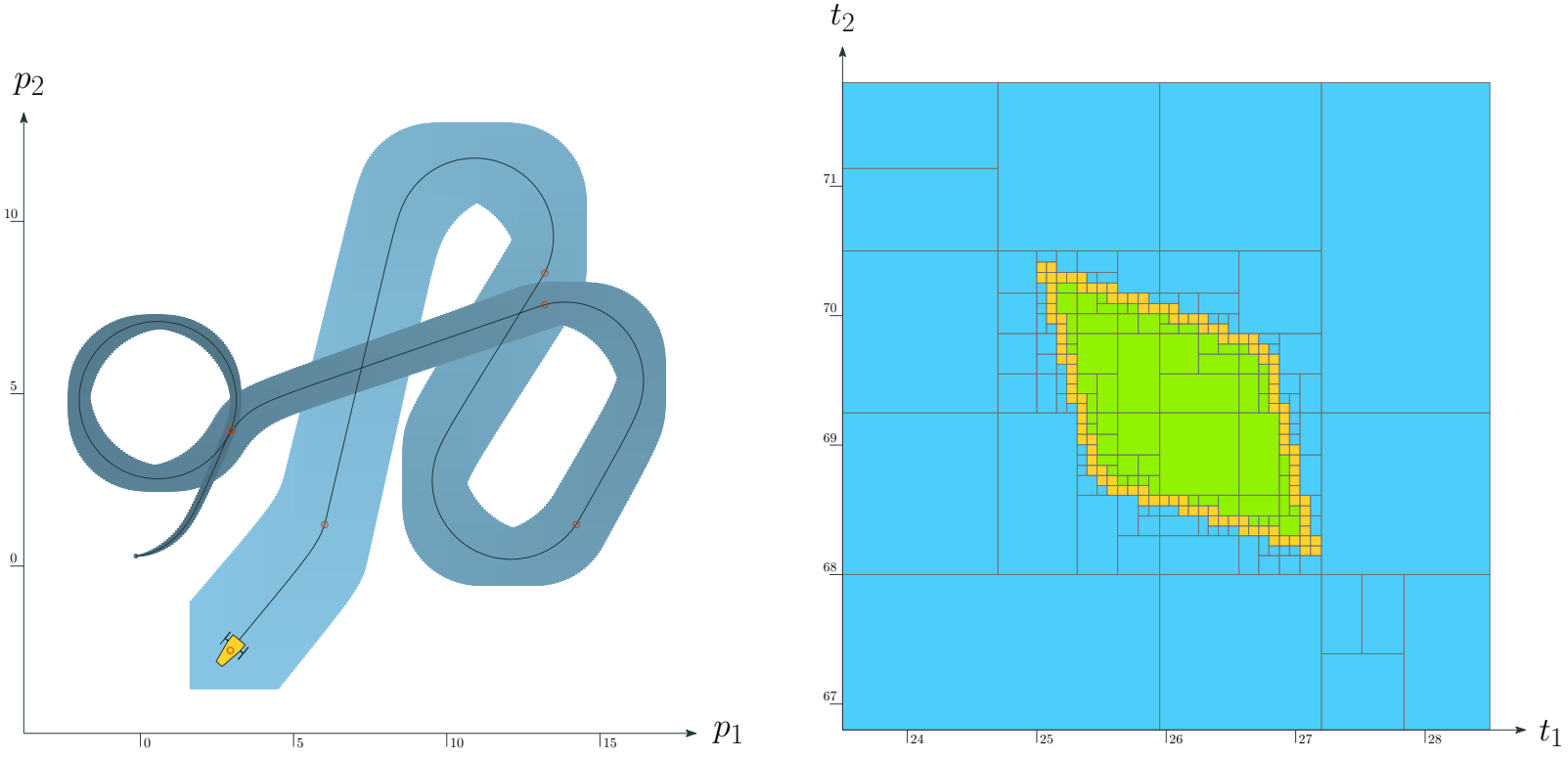

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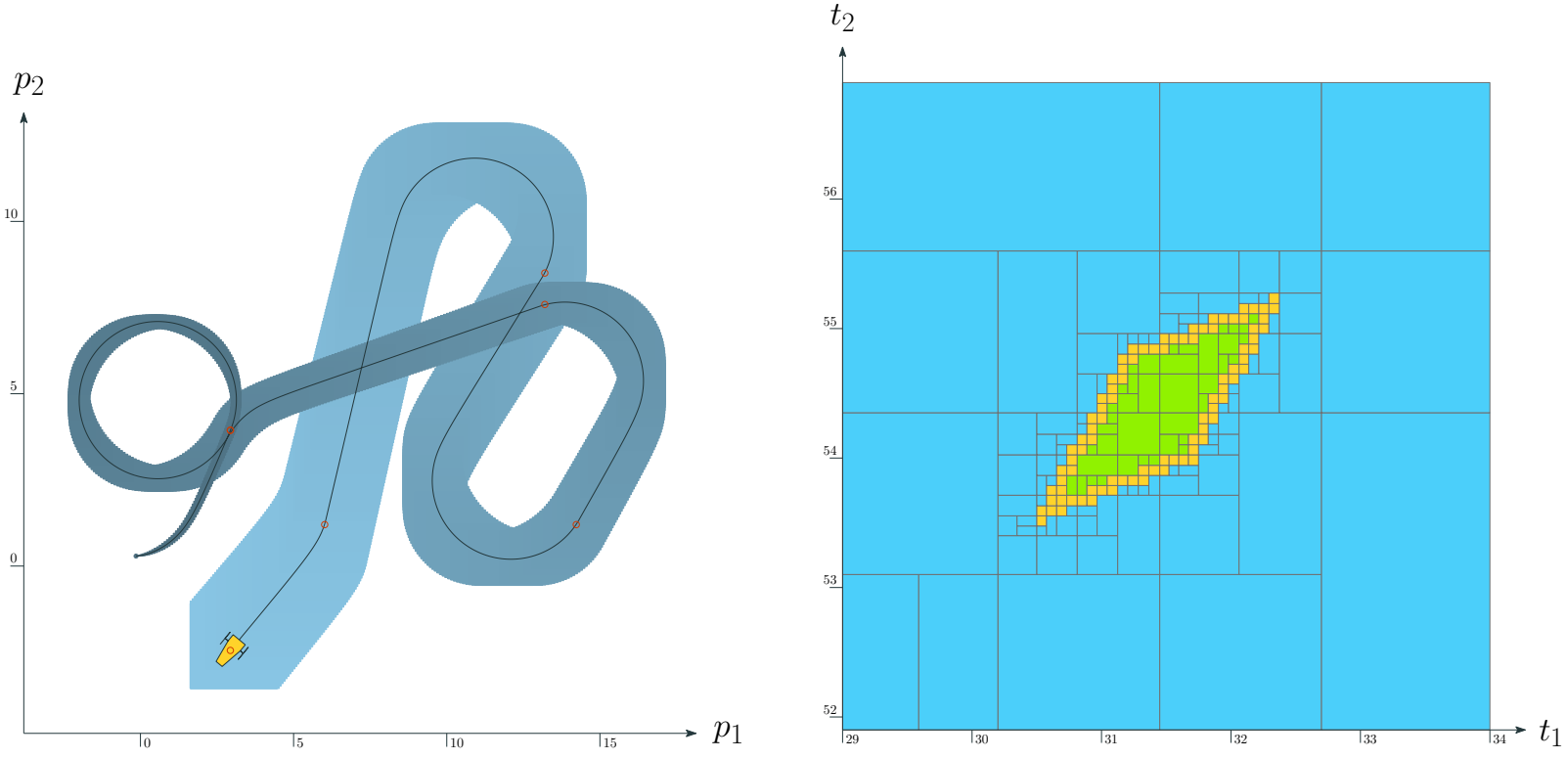

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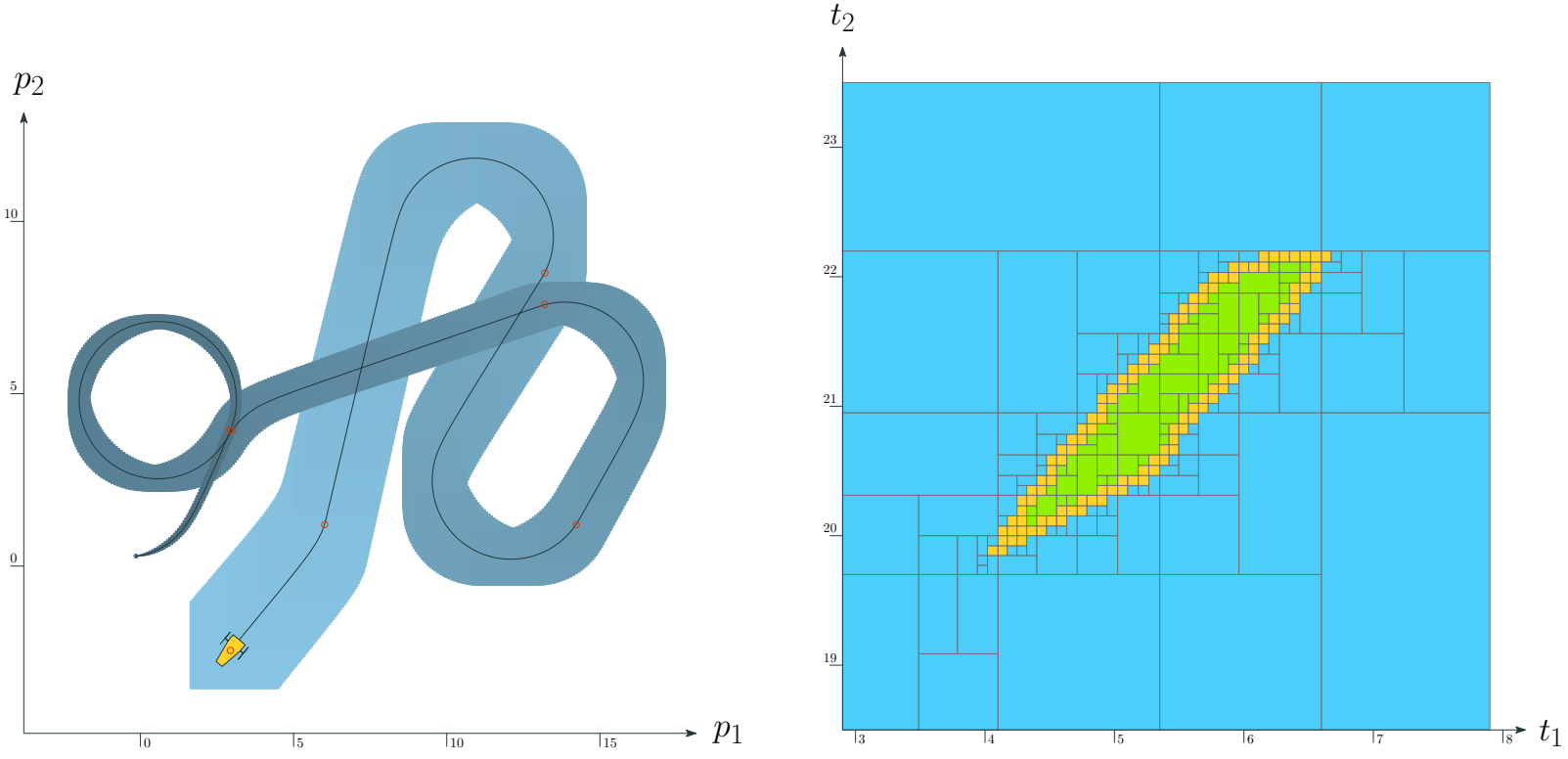

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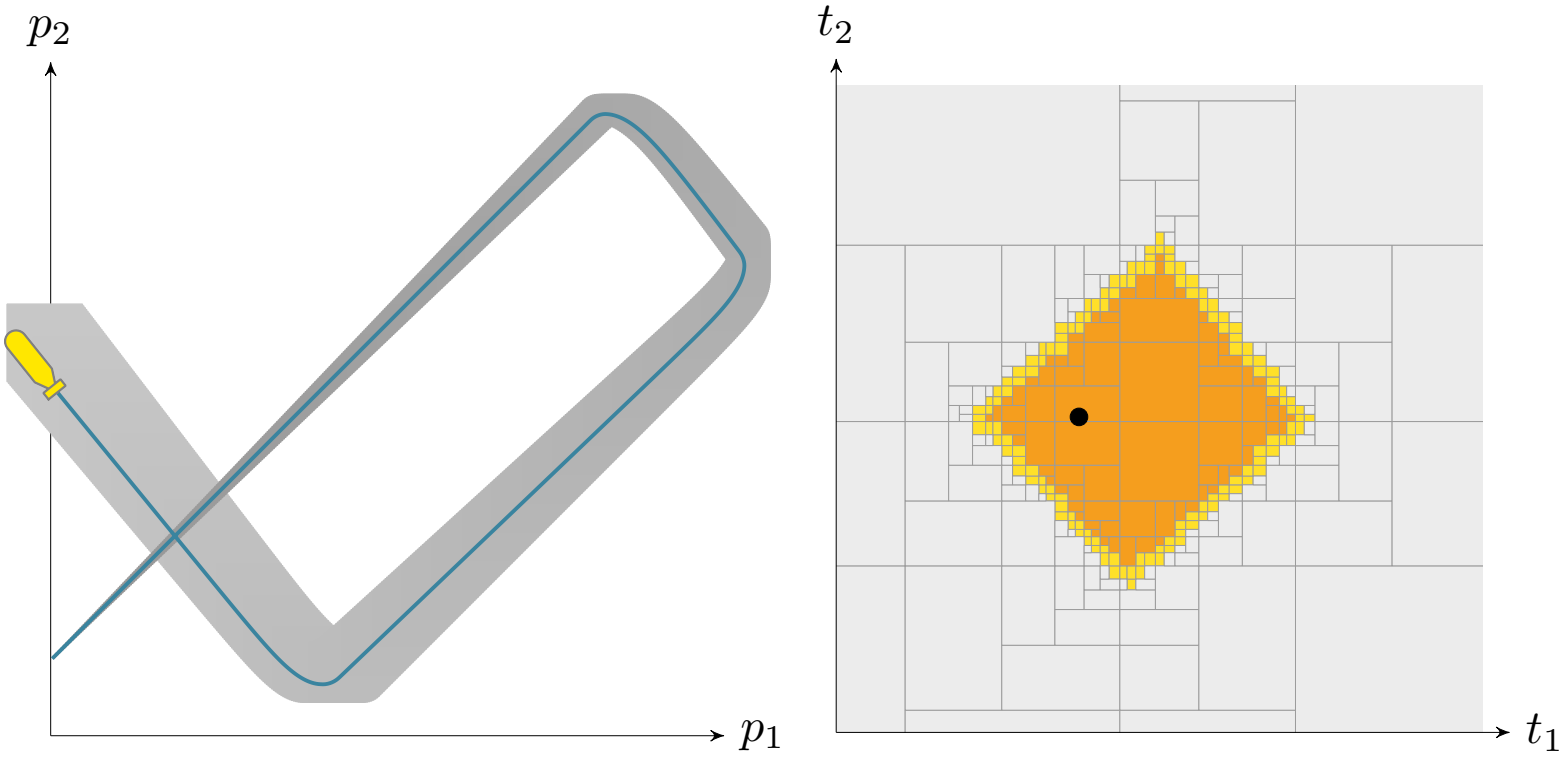
Reliable approximation of a loop set

Using a **SIVIA**-like algorithm:



Undeniable looped trajectory

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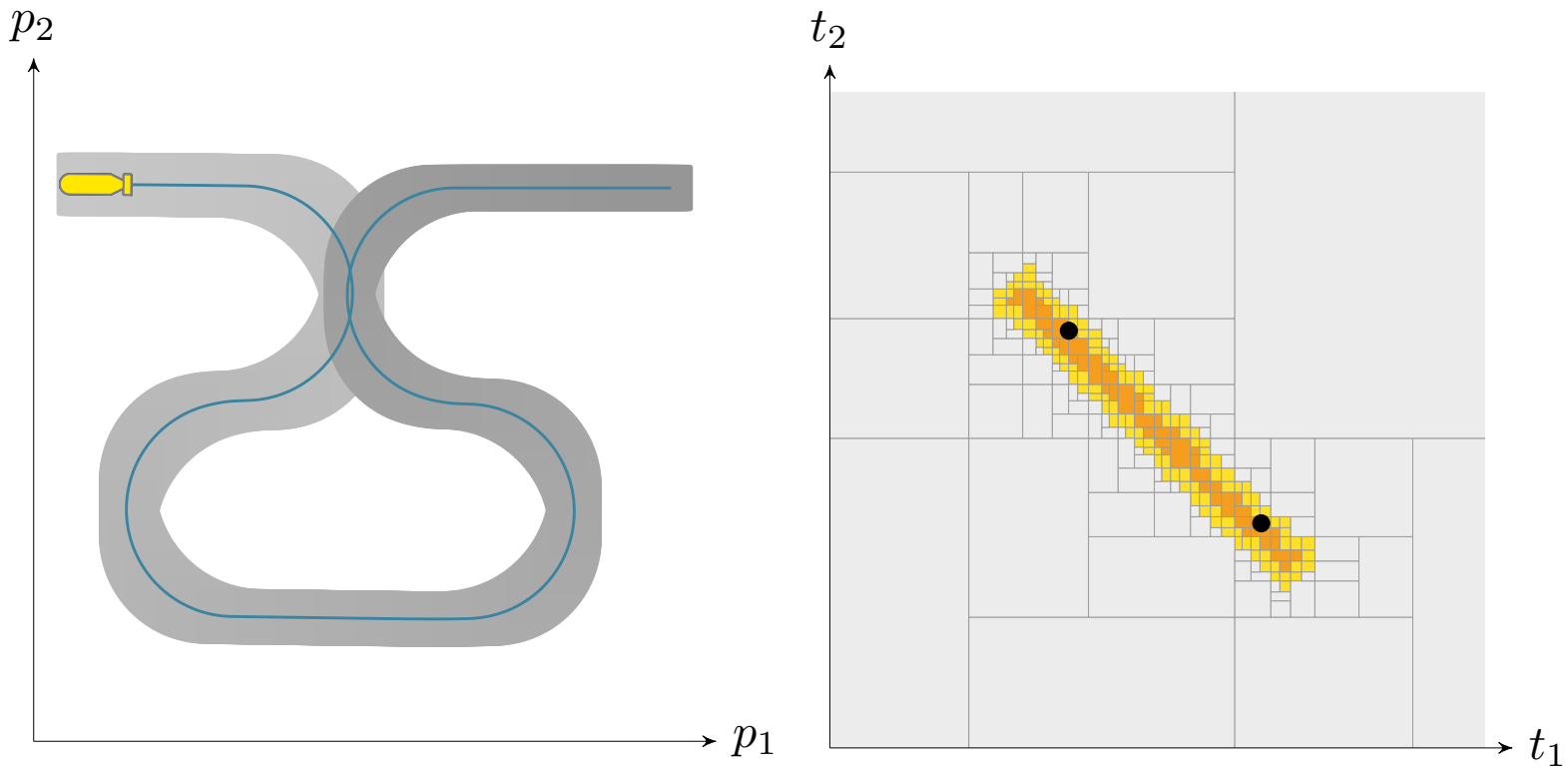

Doubtful looped trajectory

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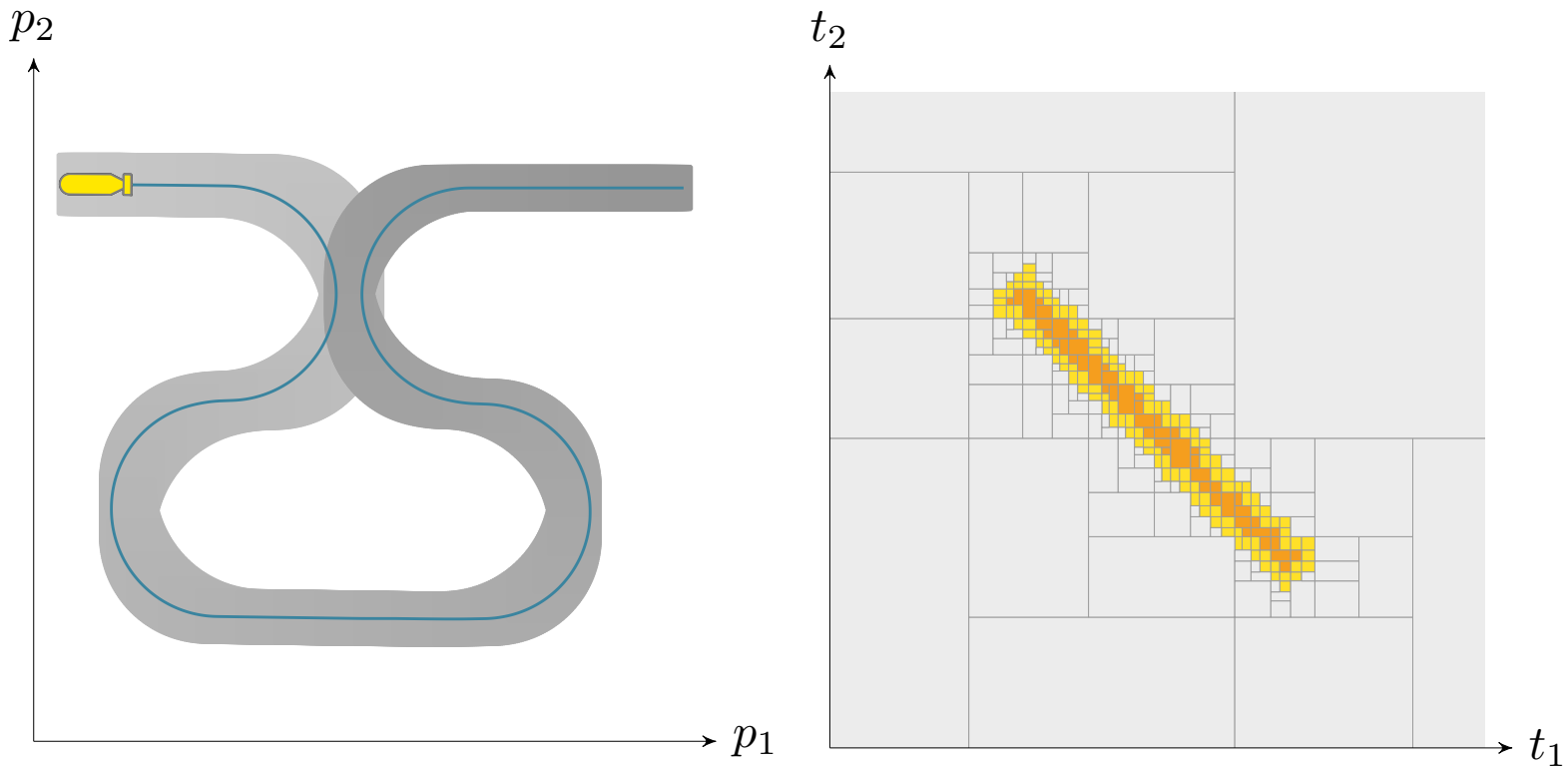

Doubtful looped trajectory

Verifying the existence of loops

Inclusion function

Simplification:

defining the actual but unknown function $\mathbf{f}^* : \mathbb{R}^2 \to \mathbb{R}^2$

$$\mathbf{f}^*(t_1, t_2) = \int_{t_1}^{t_2} \mathbf{v}^*(\tau) d\tau \tag{7}$$

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Assessed knowledge:

$[\mathbf{f}] : \mathbb{R}^2 \to \mathbb{IR}^2$ is an *interval function* of $\mathbf{f}^*$:

$$\mathbf{f}^*(t_1, t_2) \in [\mathbf{f}](t_1, t_2) = \int_{t_1}^{t_2} [\mathbf{v}](\tau) d\tau \tag{8}$$

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Verification of a loop:

$$\forall \mathbf{f} \in [\mathbf{f}], \exists \mathbf{t} \in \mathbb{T}_i \mid \mathbf{f}(\mathbf{t}) = \mathbf{0} \implies \underbrace{\exists \mathbf{t} \in \mathbb{T}_i \mid \mathbf{f}^*(\mathbf{t}) = \mathbf{0}}_{\text{loop existence proof}} \tag{9}$$

Zero verification: problem statement

Usually we cannot find exactly a zero of an uncertain function, but we can prove it exists within some domain Ω.

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Assumptions:

- known inclusion function $[\mathbf{f}] : \mathbb{IR}^n \to \mathbb{IR}^m$ of the unknown function $\mathbf{f}^* : \mathbb{R}^n \to \mathbb{R}^m$
- possibly in the form of an algorithm for computing $[\mathbf{f}]([\mathbf{t}])$
- $n = m = 2$
- Ω possibly made of a union of finitely many boxes in $\mathbb{IR}^2$

$\longrightarrow$ need to isolate and verify zeros of $\mathbf{f}^*$

Powerful topological degree

Topological degree $\deg(\mathbf{f}^*, \Omega)$:

- unique integer assigned to $\mathbf{f}^*$ and a compact set $\Omega \subset \mathbb{R}^n$ such that $\mathbf{f}^*(\mathbf{t}) \neq \mathbf{0}$ for all $\mathbf{t} \in \partial\Omega$

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Most important property of it:

$$\deg(\mathbf{f}^*, \Omega) \neq 0 \implies \exists \mathbf{t} \in \Omega \mid \mathbf{f}^*(\mathbf{t}) = \mathbf{0} \tag{10}$$

- Topological degree theory and applications
  Y. J. Cho, Y. Q. Chen *Mathematical Analysis and Applications*, 2006
- Degree theory in analysis and applications
  I. Fonseca, W. Gangbo *Oxford lecture series*, 1995
- A set of axioms for the degree of a tangent vector field on differentiable manifolds
  M. Furi, M. P. Pera, M. Spadini *Fixed Point Theory and Applications*, 2010

Powerful topological degree

Assets of topological degree:

- can be computed in case where only an inclusion function $[\mathbf{f}]$ of $\mathbf{f}^*$ is given.

  - Effective topological degree computation based on interval arithmetic
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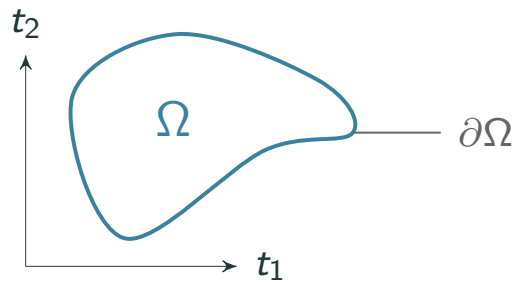

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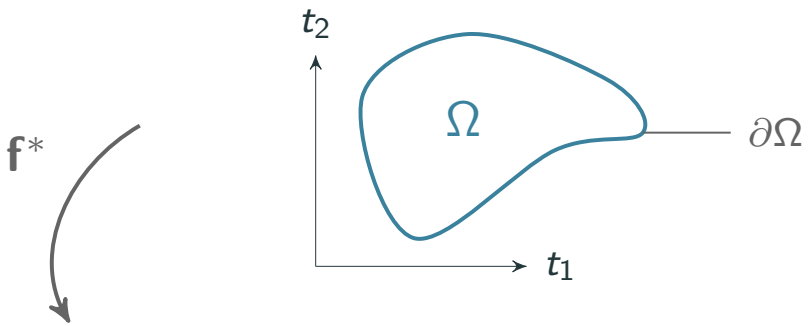
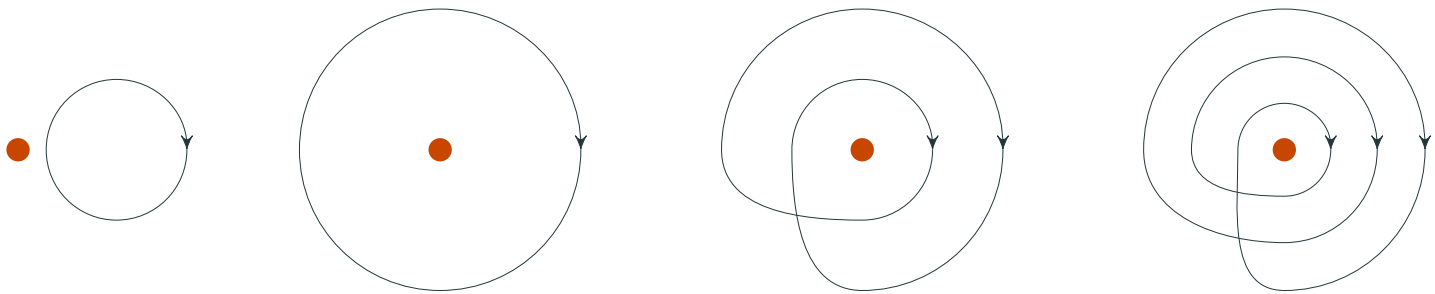
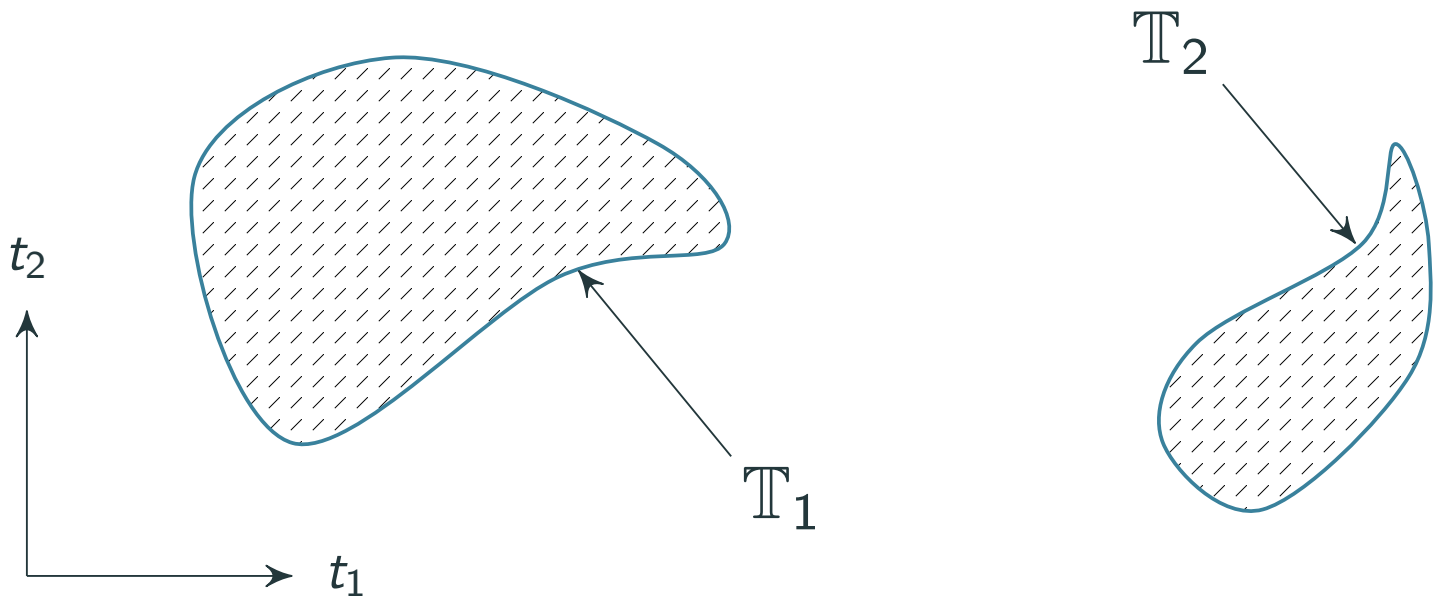




$\deg = 0$ $\qquad$ $\deg = 1$ $\qquad$ $\deg = 2$ $\qquad$ $\deg = 3$

Outer approximation of a set $\mathbb{T}$ with SIVIA

Consider $\mathbb{T} \subset \mathbb{R}^2$ in which we want to find zeros of $\mathbf{f}^*$.



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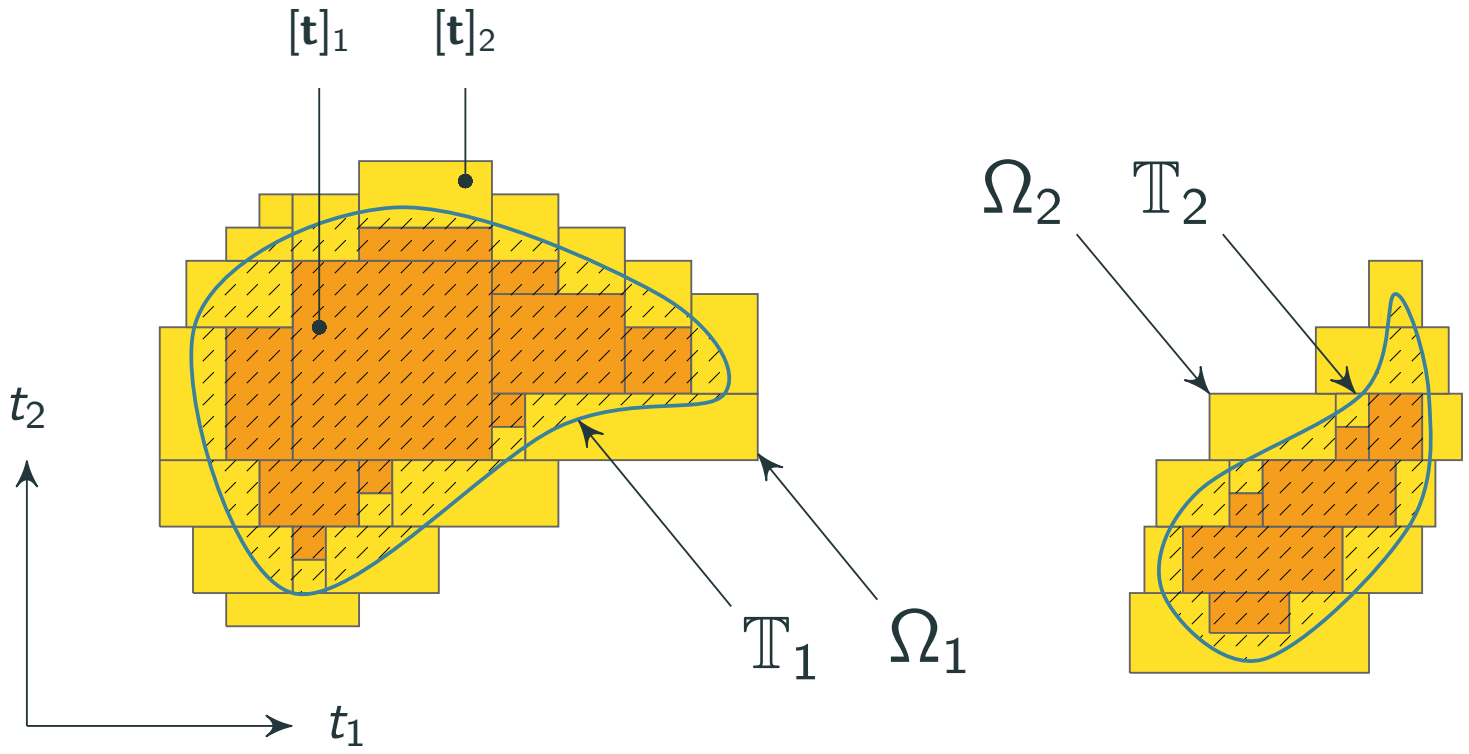

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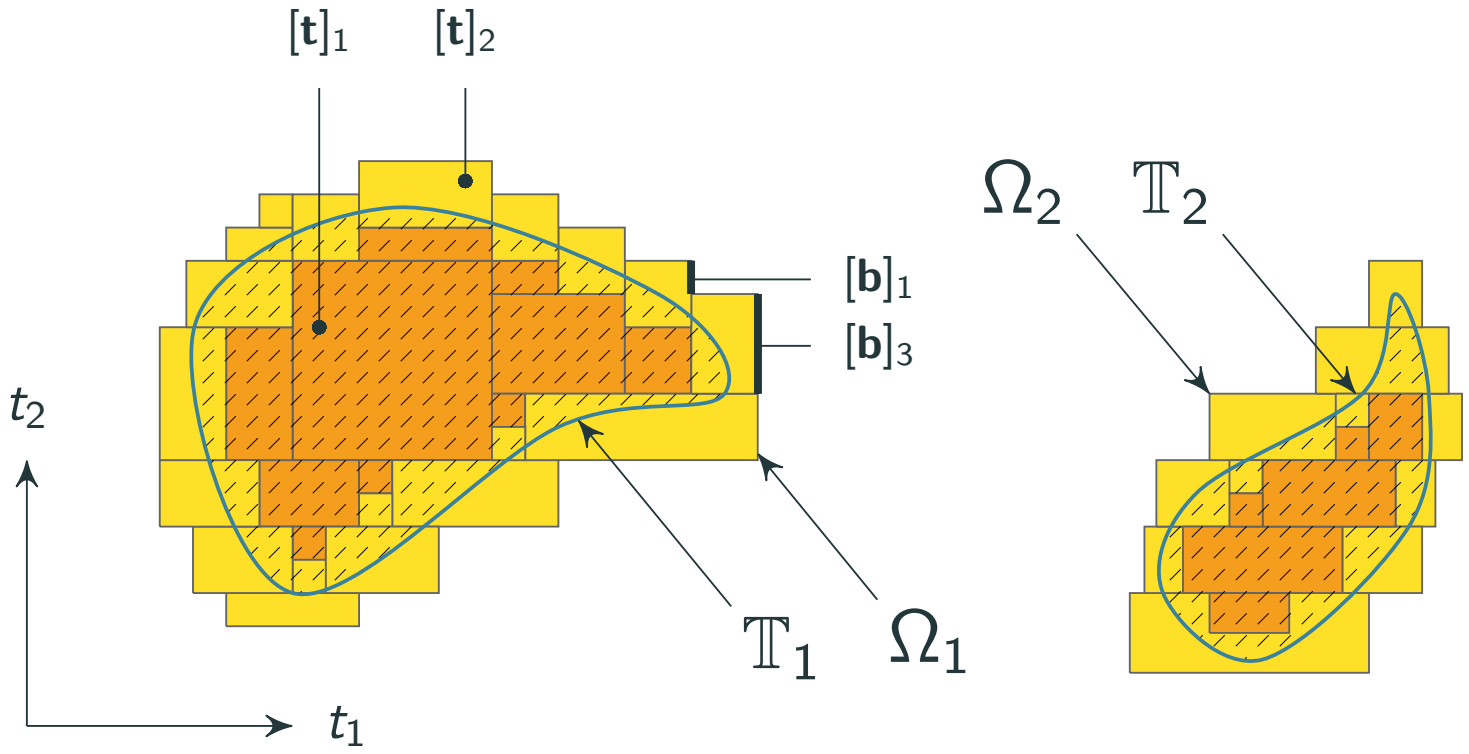

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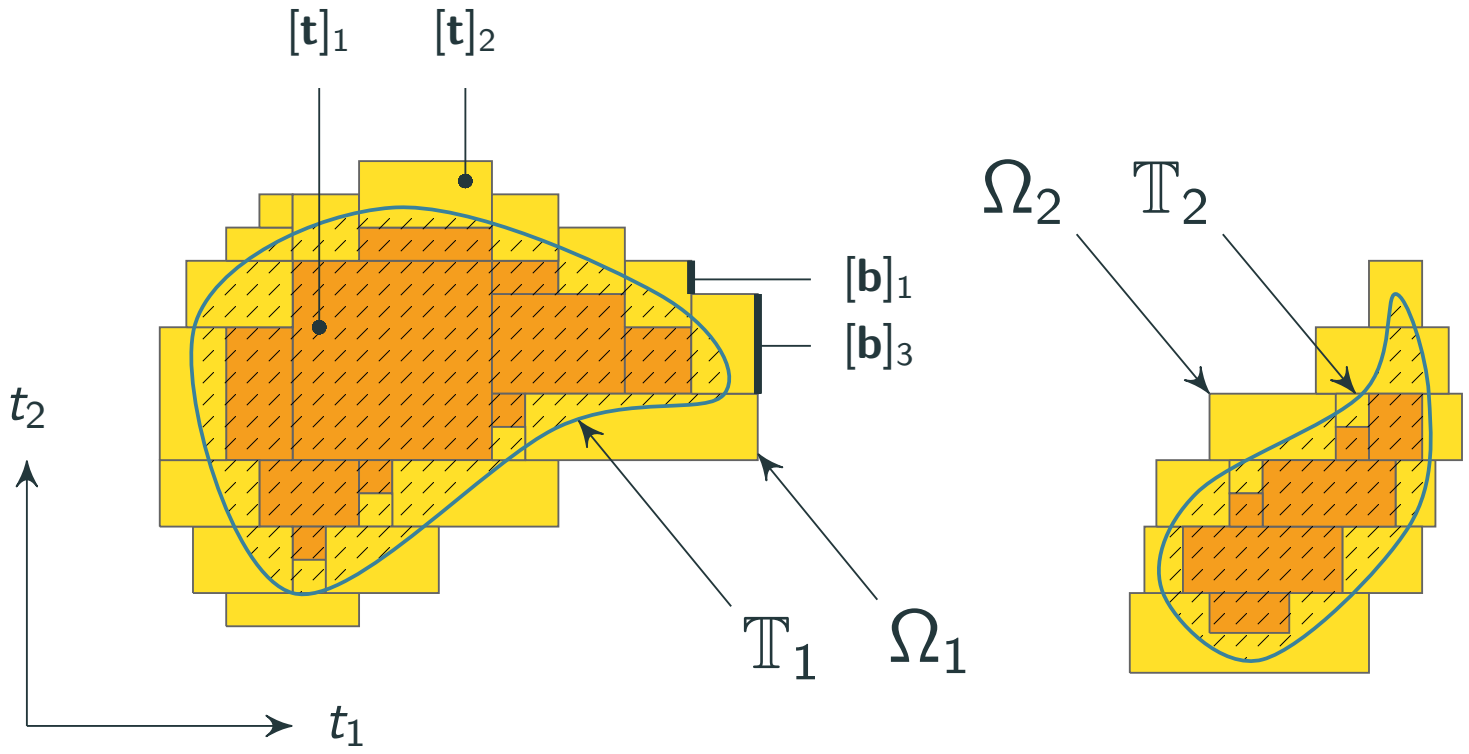
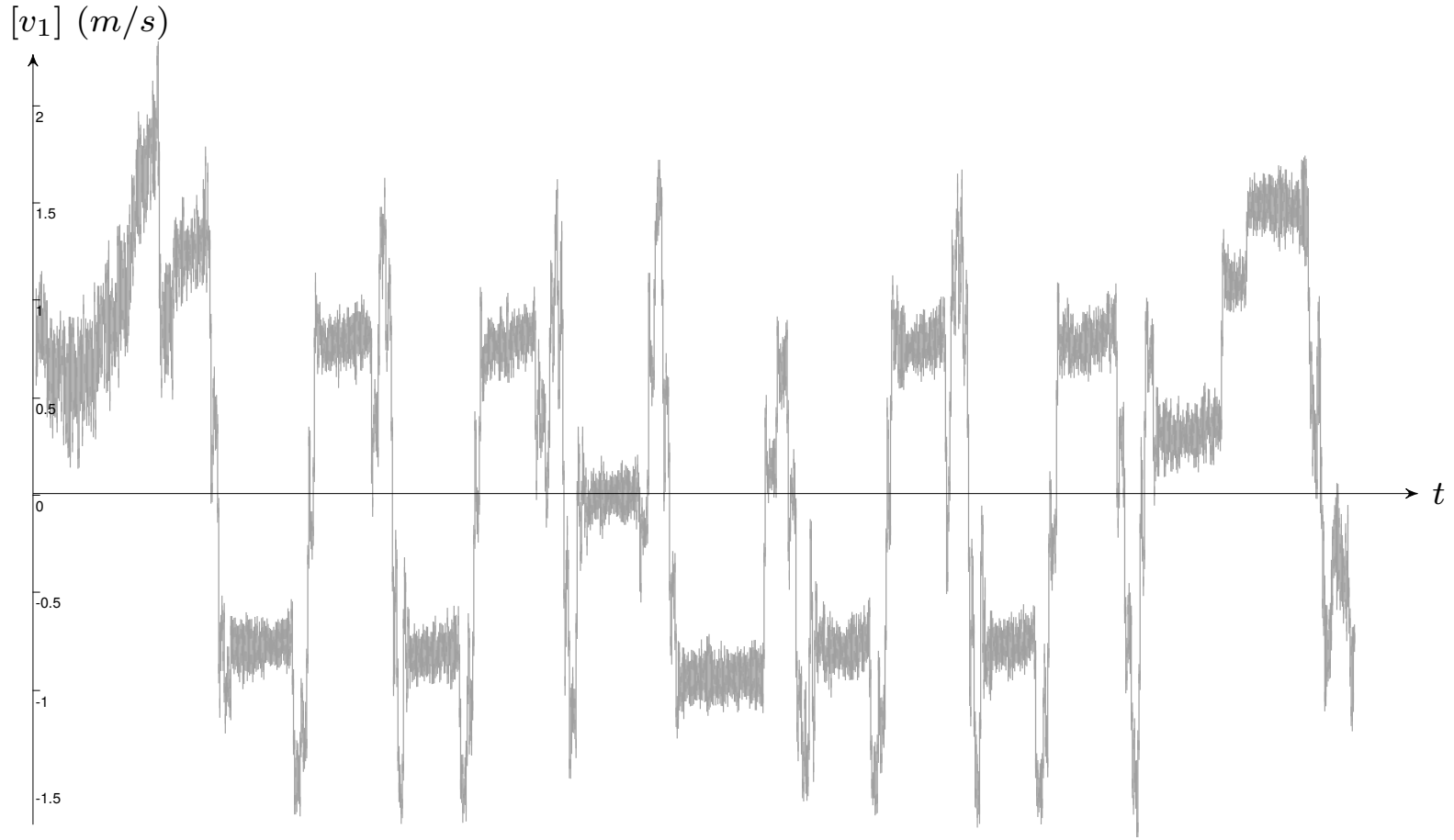


Outer set has the properties required for Ω: $\mathbf{f}^*(\mathbf{t}) \neq \mathbf{0},\ \forall \mathbf{t} \in \partial\Omega$

Verifying loops on actual datasets

Enclosed velocities

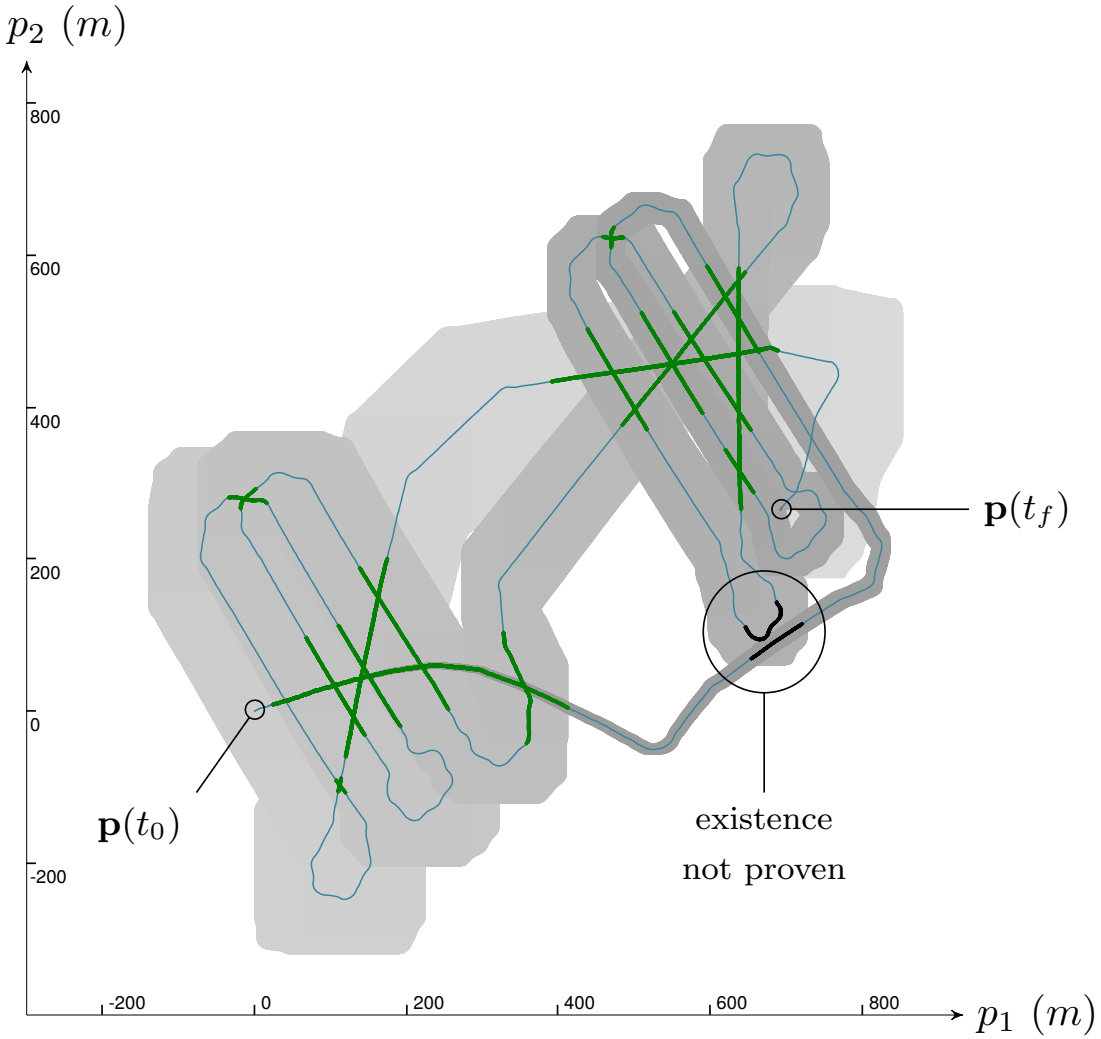
Tube $[\mathbf{v}](\cdot)$ built from sensor data:
(DVL for velocity + AHRS for attitude)



East speed velocity tube $[v_1](\cdot)$.

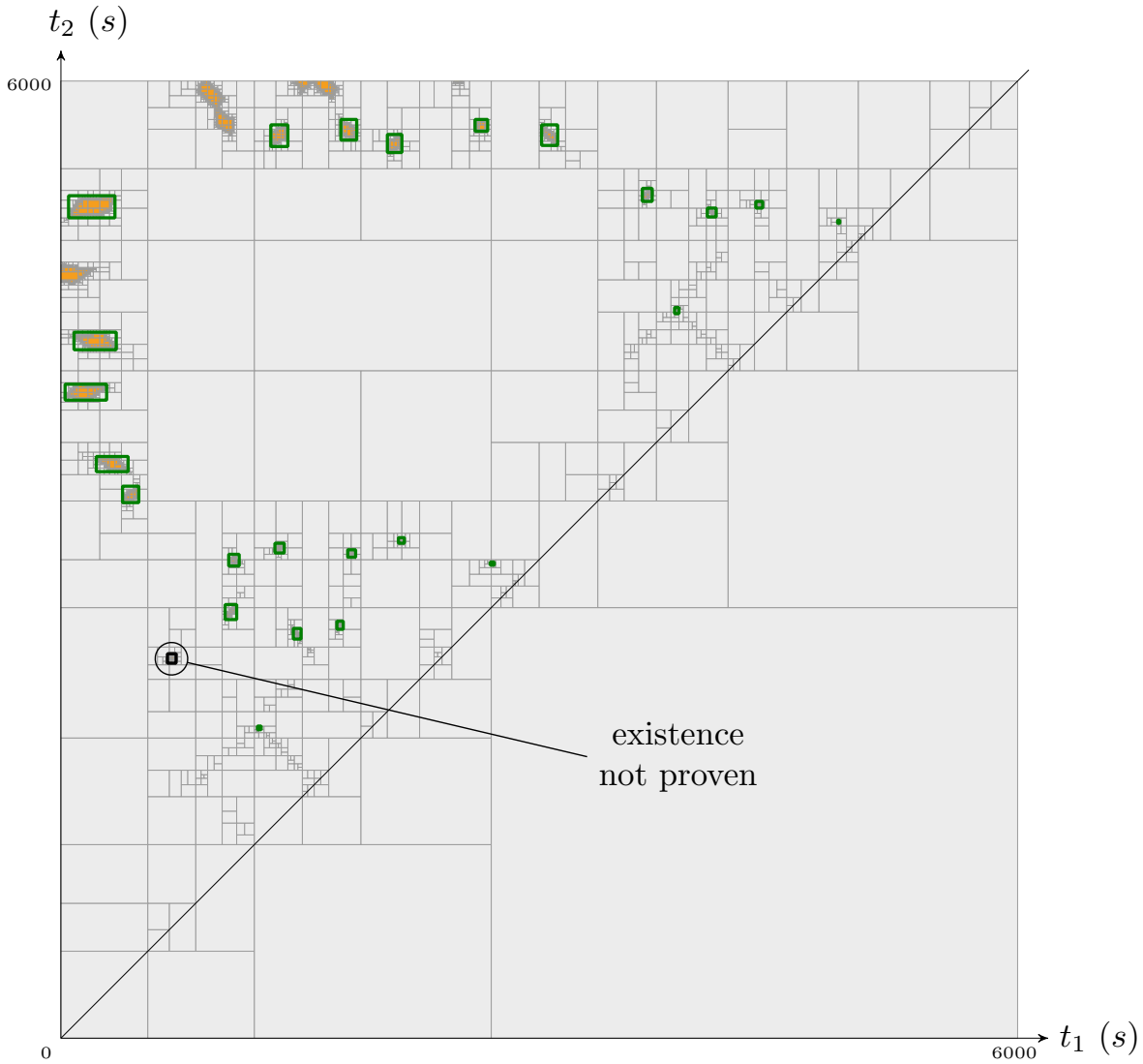
Redermor's mission

Guaranteed enclosure of the trajectory:



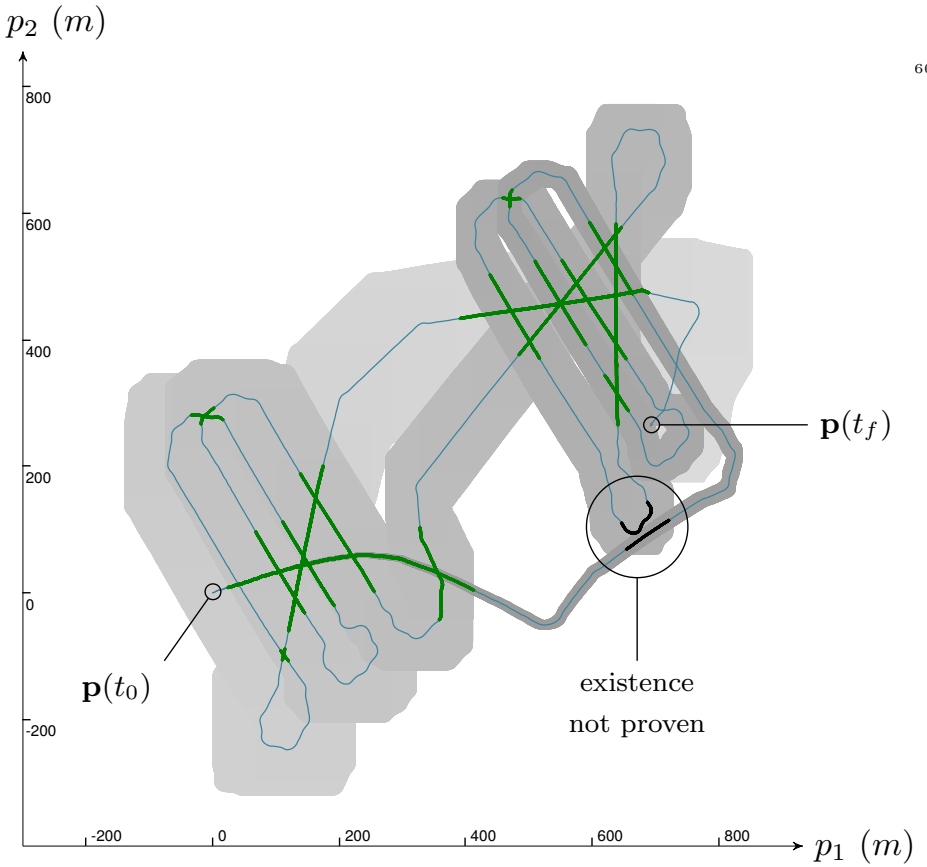
Tube $[\mathbf{p}](\cdot)$ *enclosing the feasible trajectories of Redermor AUV.*

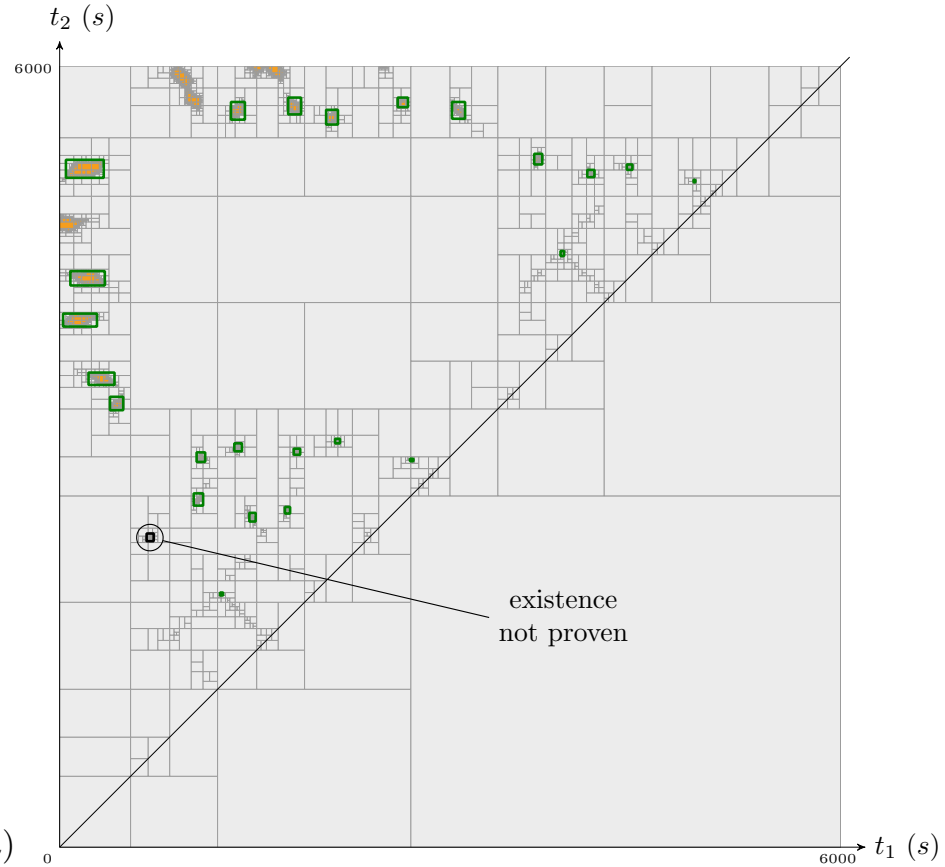
t-plane of the mission: $\mathbb{T} = \{(t_1, t_2) \mid \mathbf{0} \in [\mathbf{f}](t_1, t_2), t_1 < t_2\}$



t-plane corresponding to Redermor's mission.

Redermor's mission: overview and results





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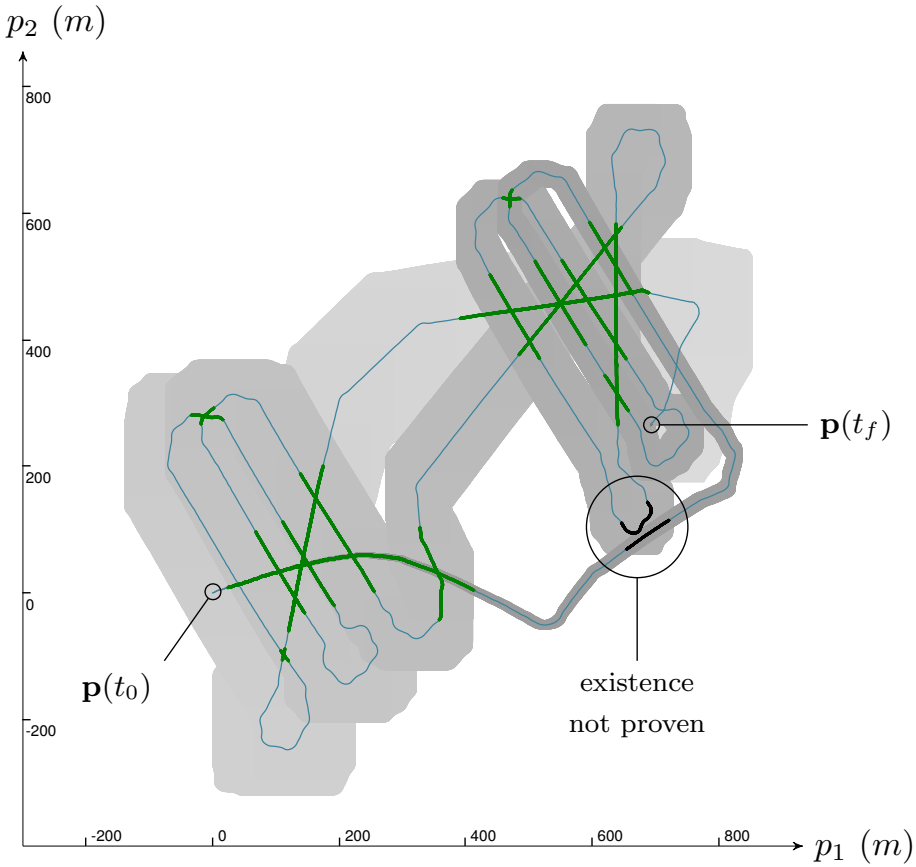

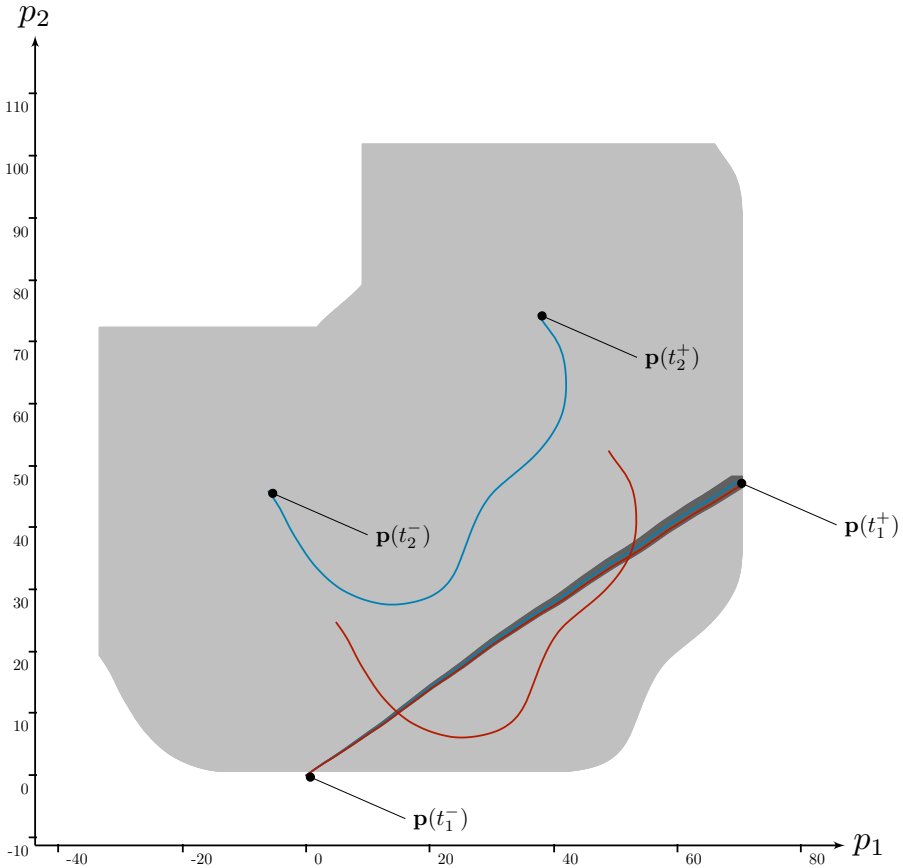

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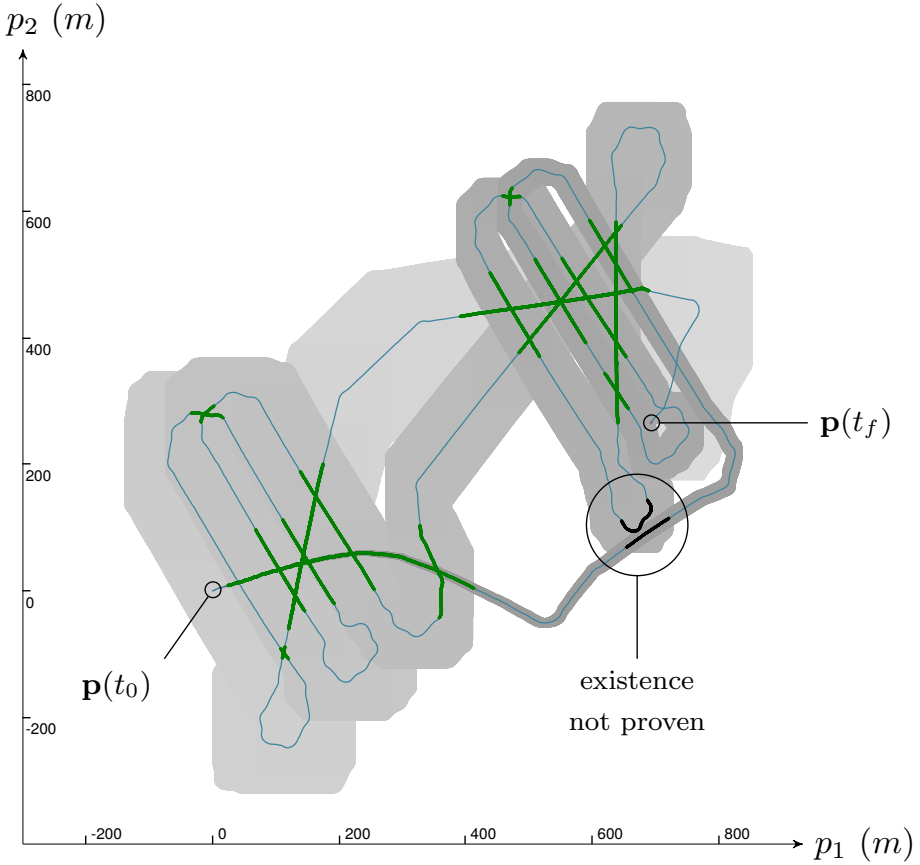

Loop proof number

Without uncertainties:

$$\lambda^* = \#\{\mathbf{t} \mid \mathbf{f}^*(\mathbf{t}) = \mathbf{0}, t_1 < t_2\}$$

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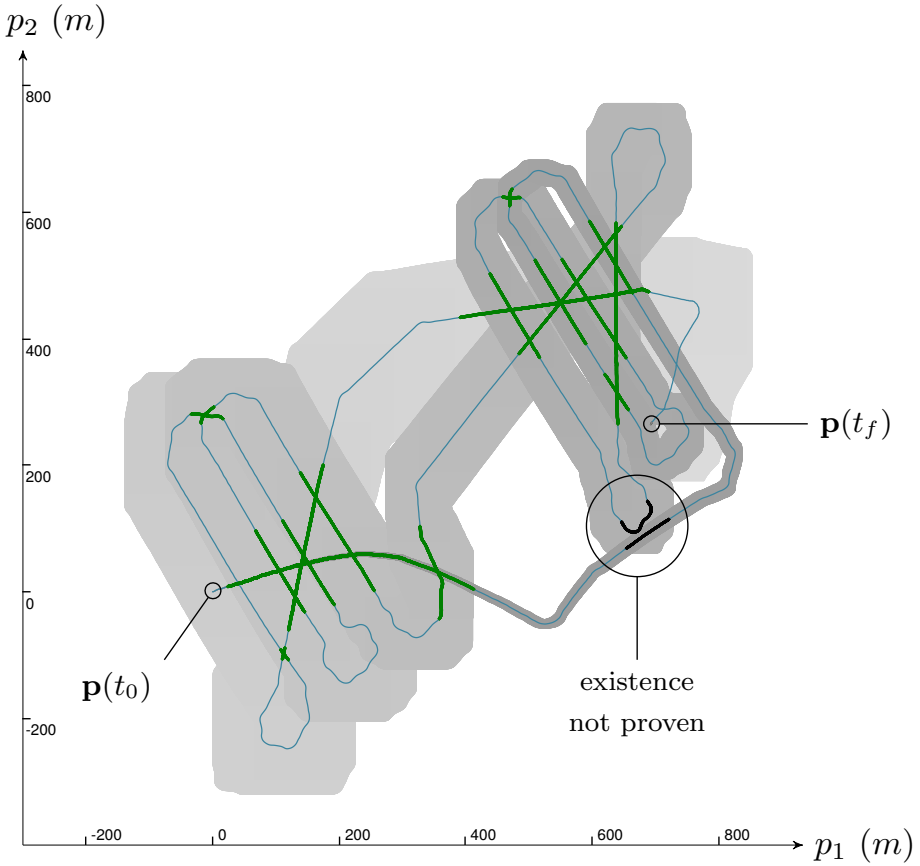

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Newton operator test: $\lambda_{\mathcal{N}} = 14$

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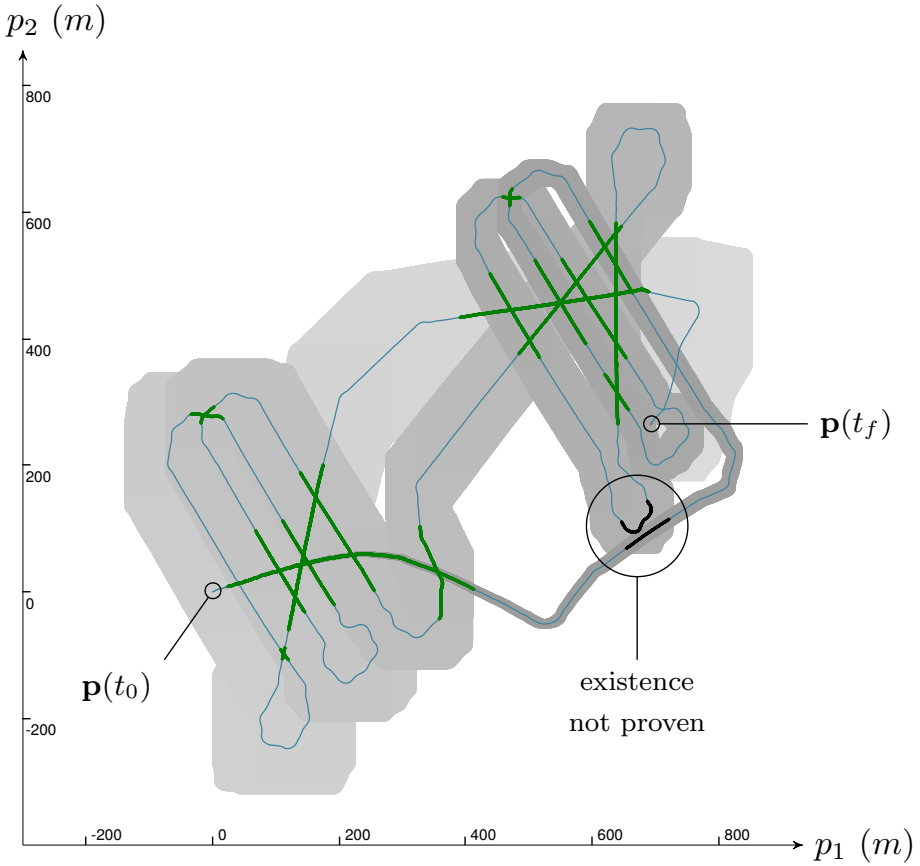

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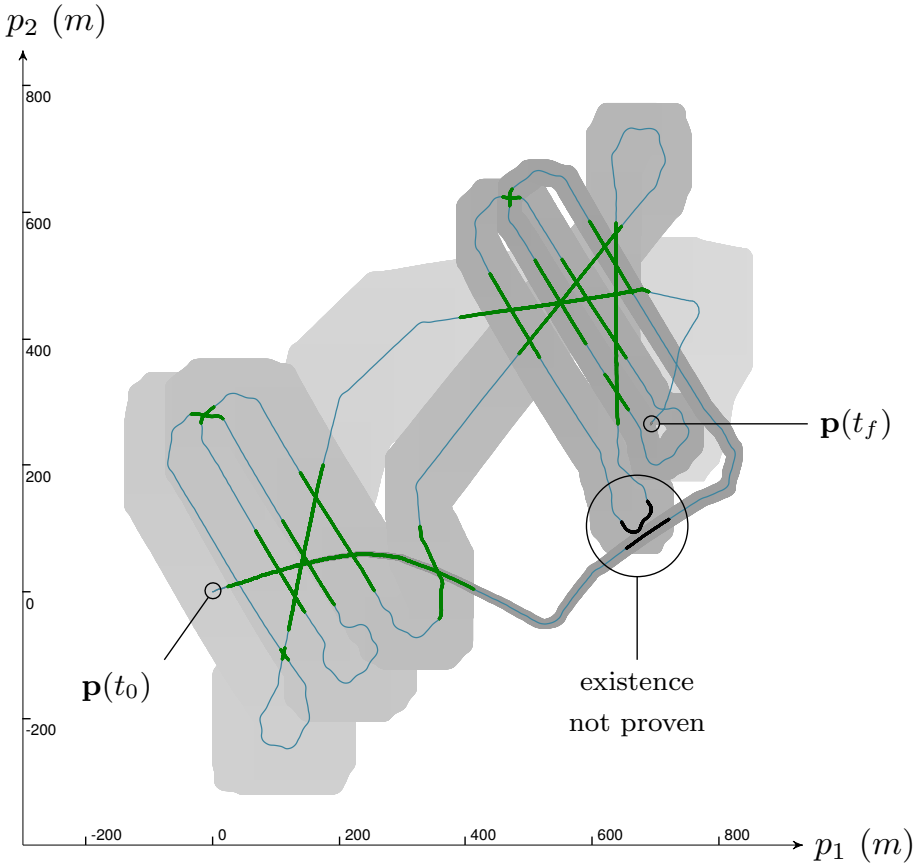

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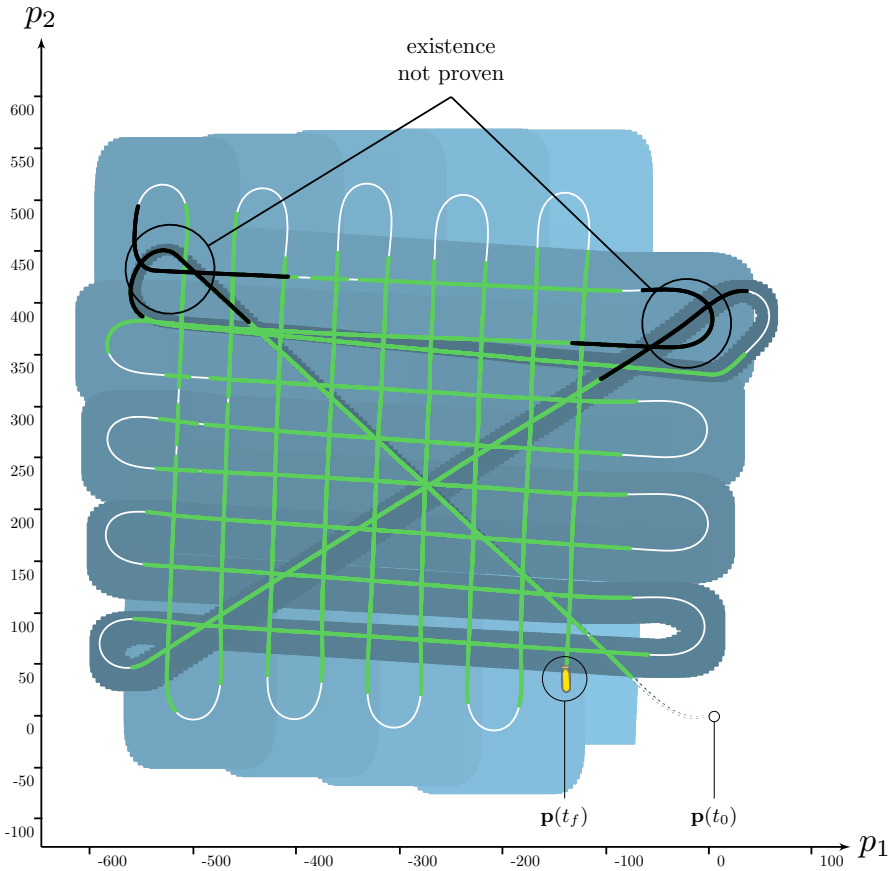
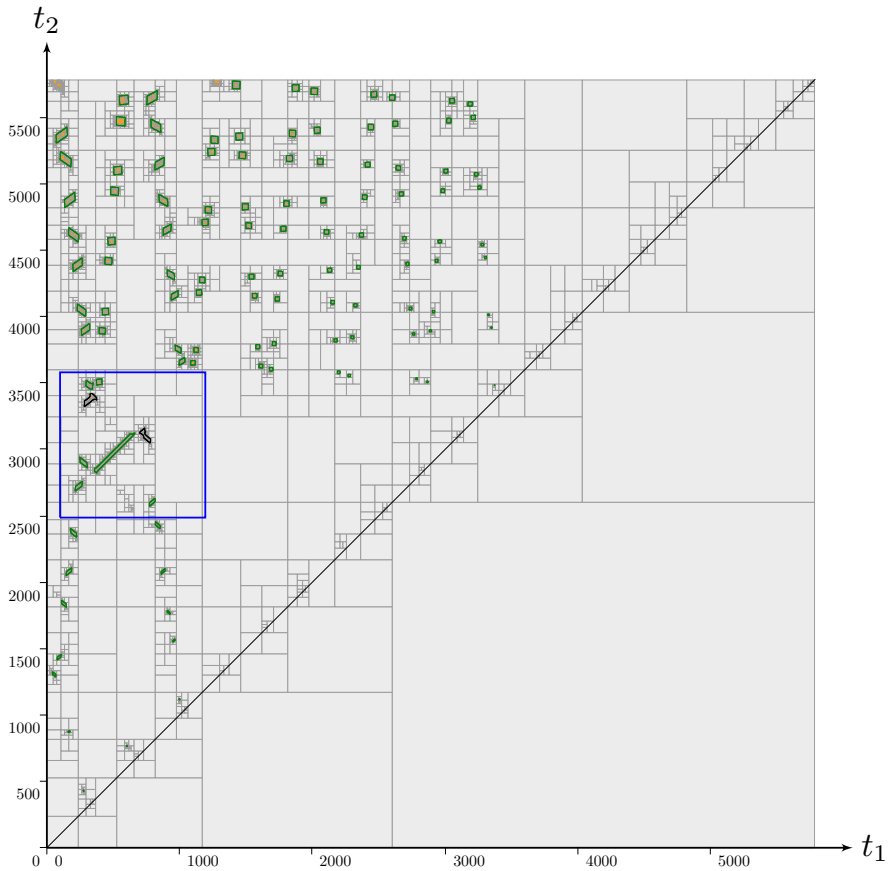
Truth: $\lambda^* = 24$

Another experiment: *Daurade*'s mission





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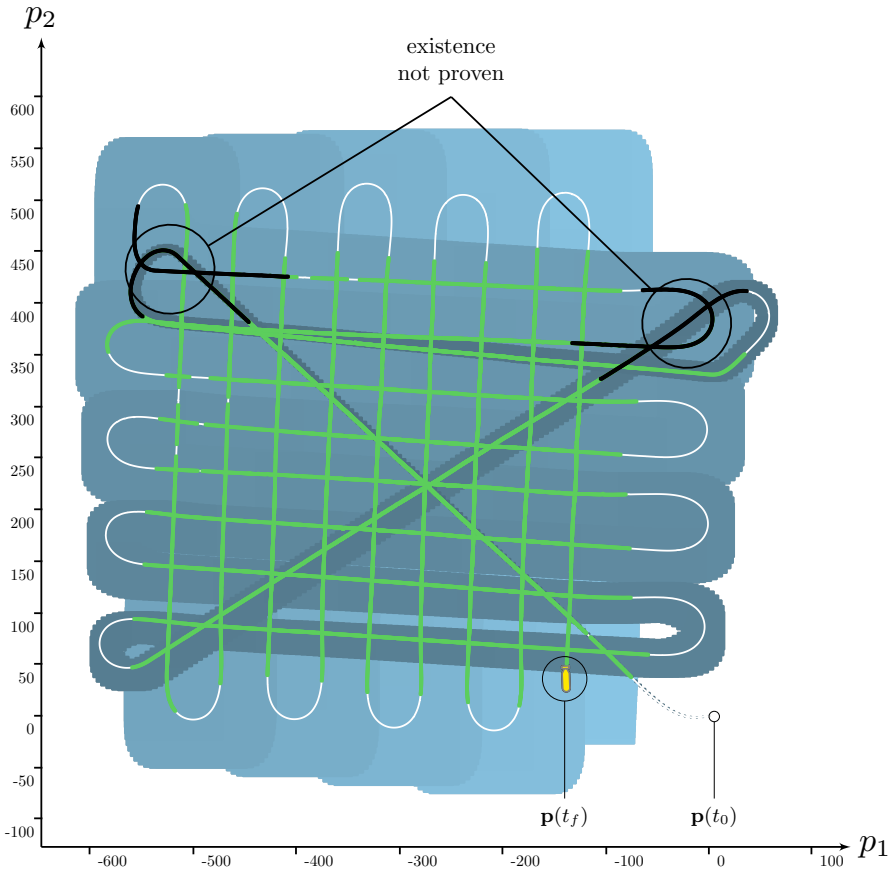

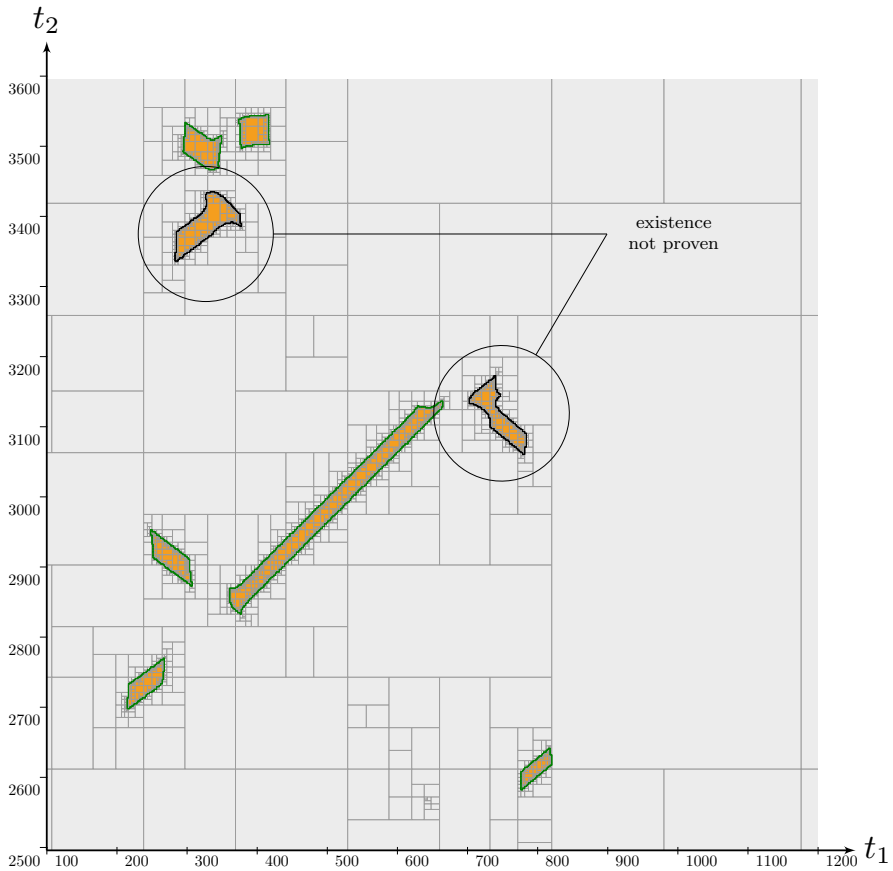

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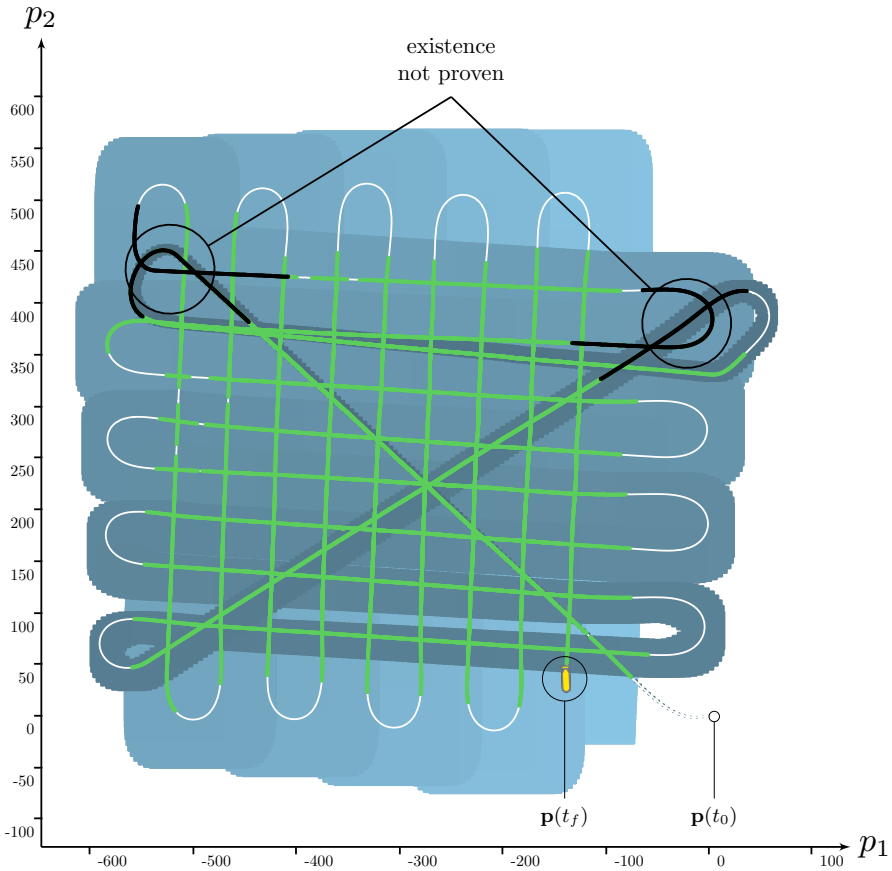

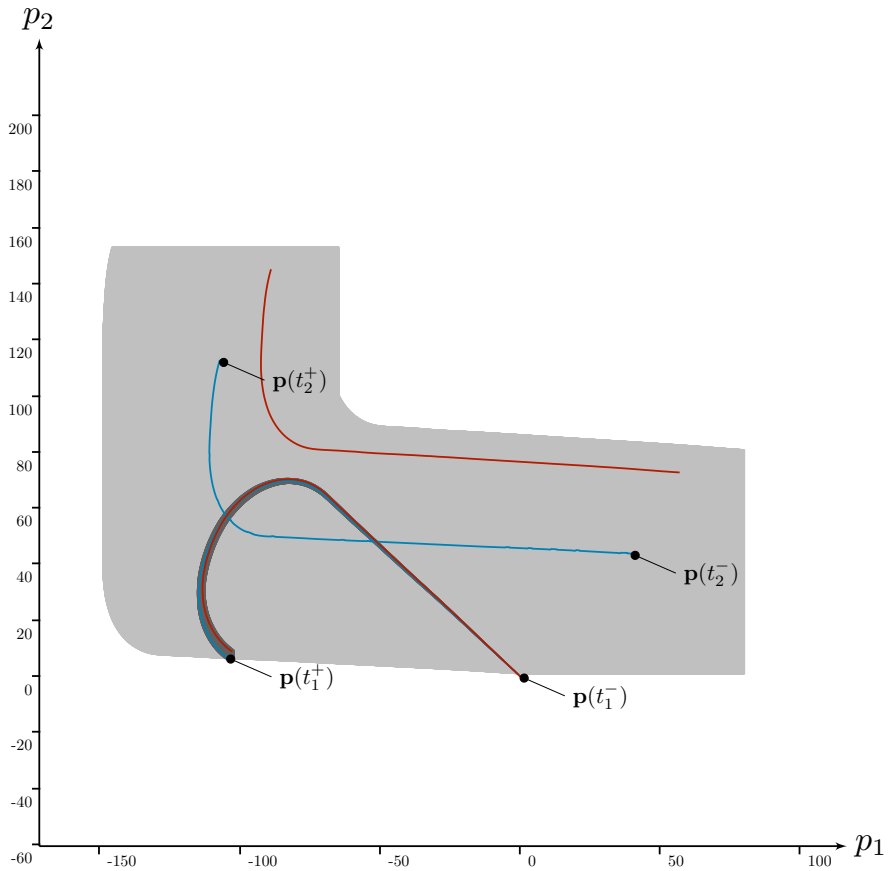

Proving loops: conclusion

Loop proof $\Leftrightarrow$ **verified existence of a 0** of an uncertain function:

- context where the exact values of the function are not known
- have to deal with a reliable approximation of it

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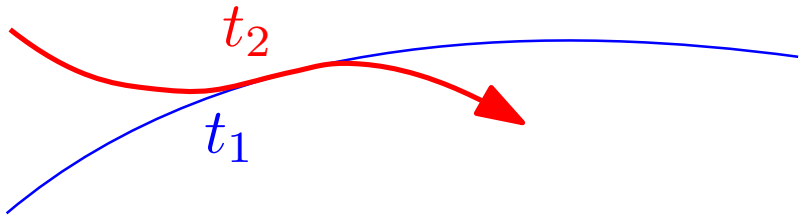

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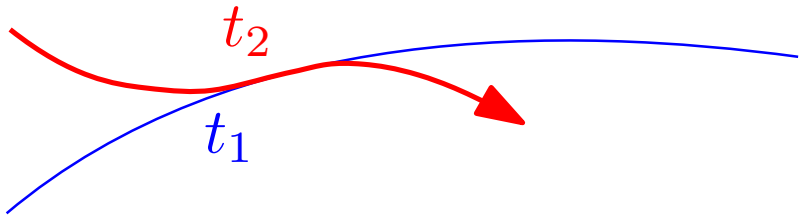


Proving the existence of loops in robot trajectories

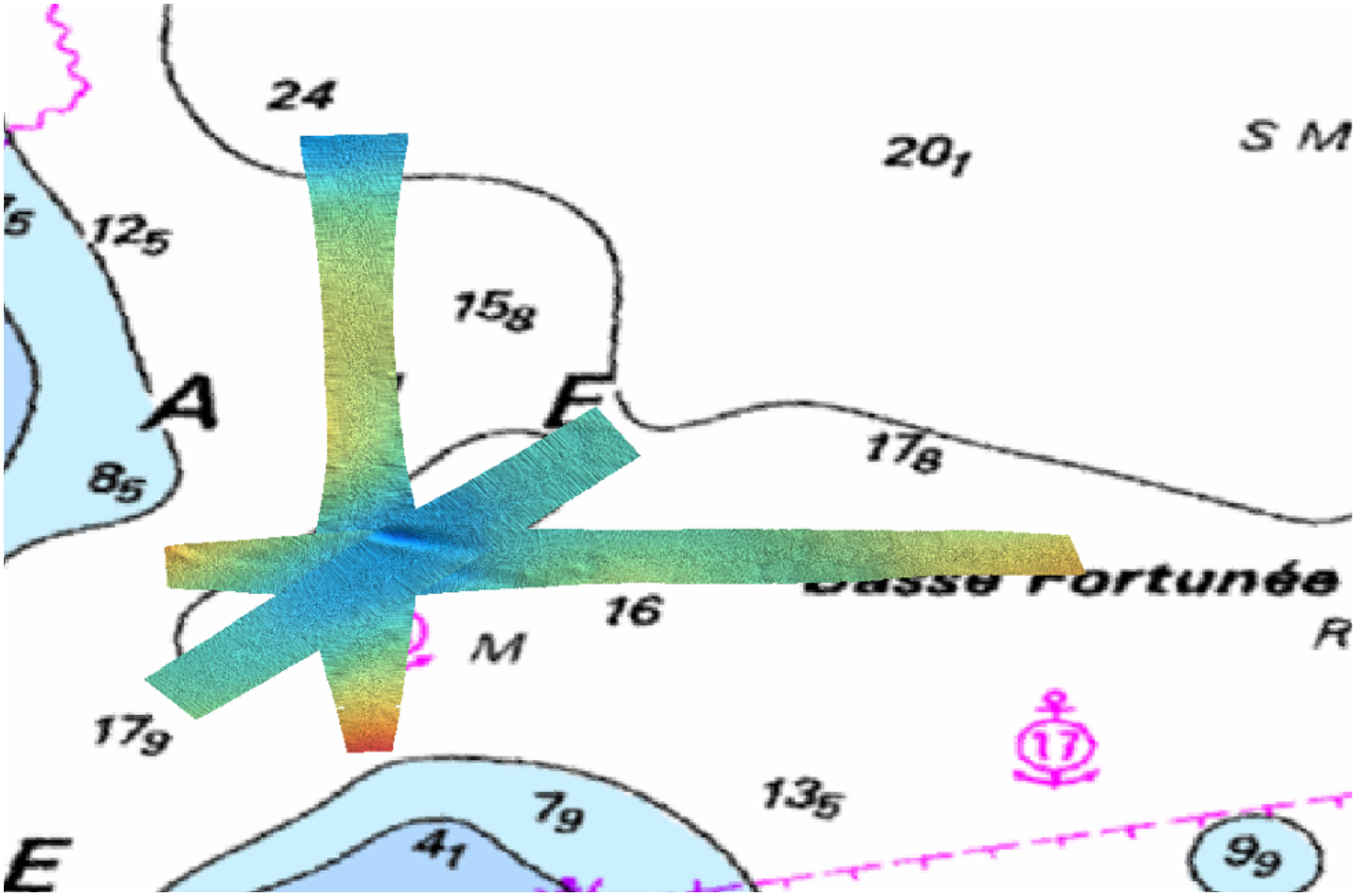
The International Journal of Robotics Research, 2018

Simon Rohou, Peter Franek, Clément Aubry, Luc Jaulin

Robot localization

Ground observations: crossing acquisitions

Goal: use bathymetric information for localization purposes



Crossed acquisition tracks (looped trajectories), in case of positioning drift.

Ground observations: crossing acquisitions

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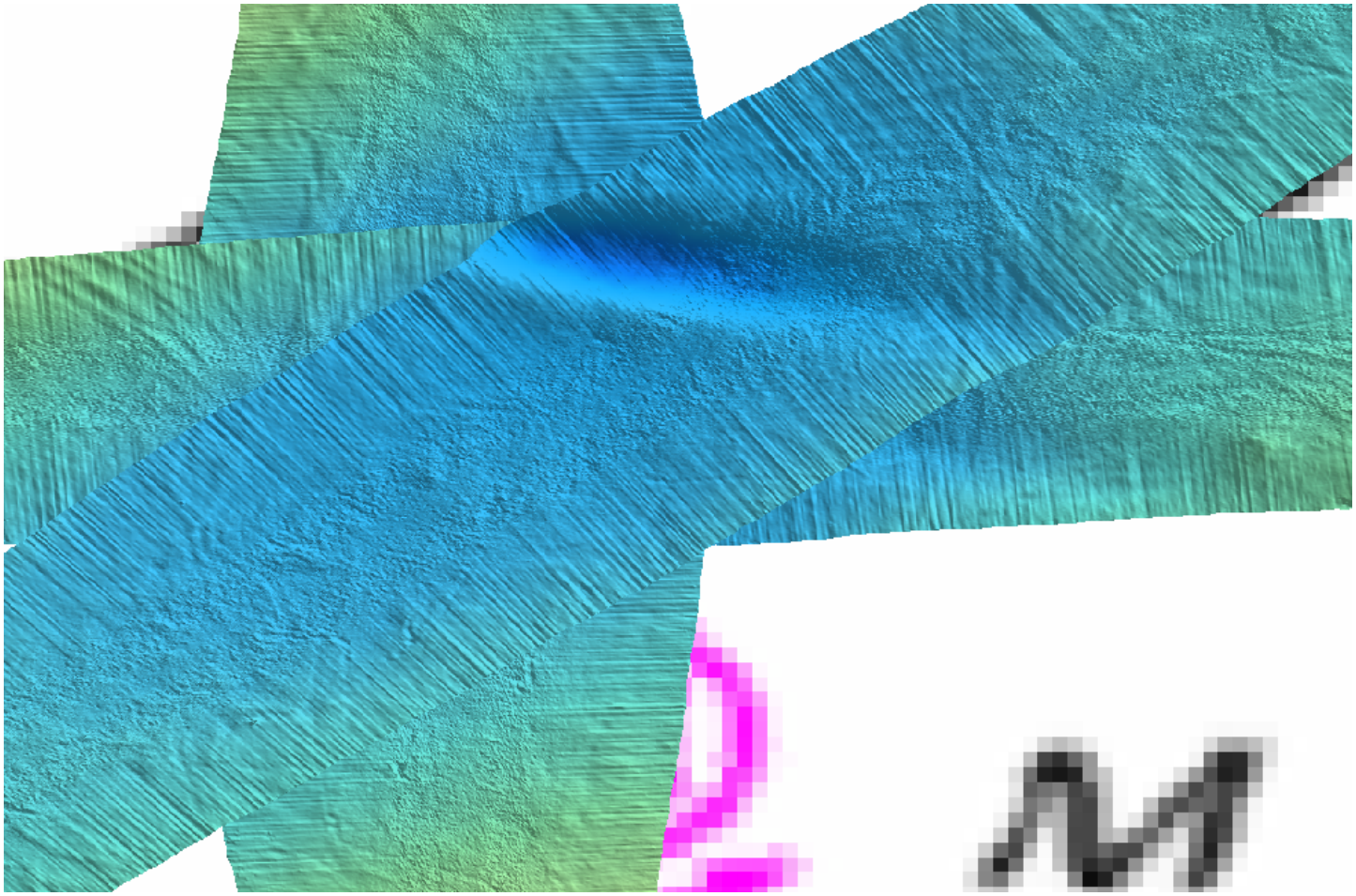
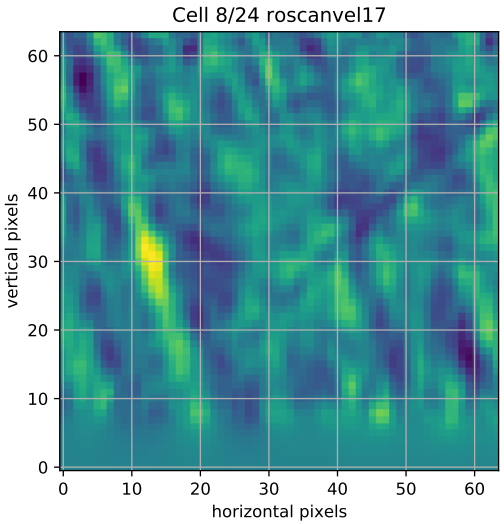
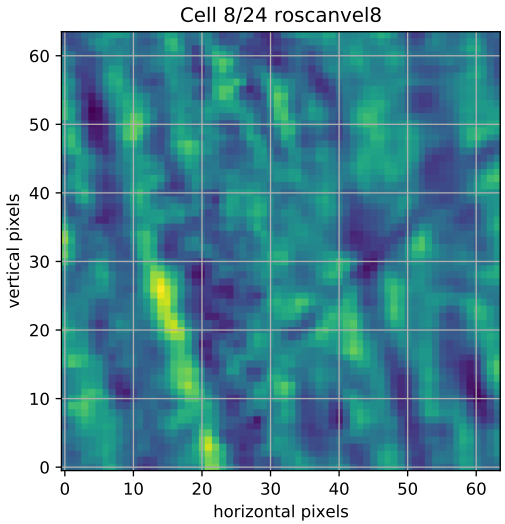
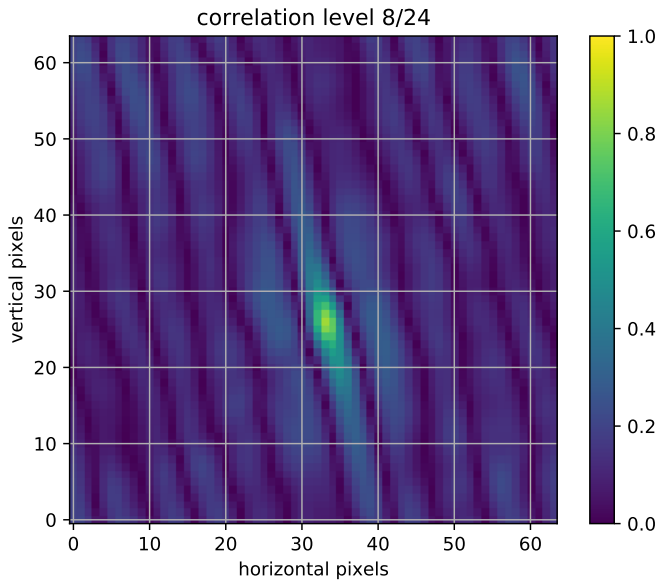


Crossed acquisition tracks (looped trajectories), in case of positioning drift.

Correlation of two ground observations







- Correlation: 0.84

  $d_1 = 0.58\text{m}$

  $d_2 = \text{-}2.53\text{m}$

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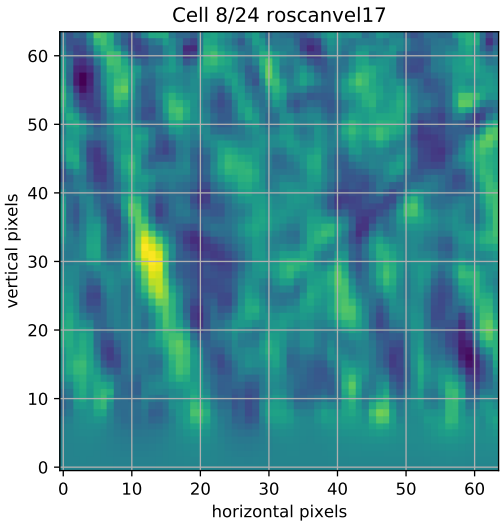

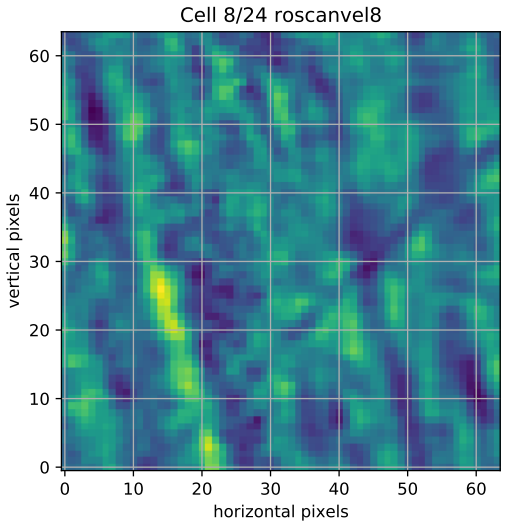

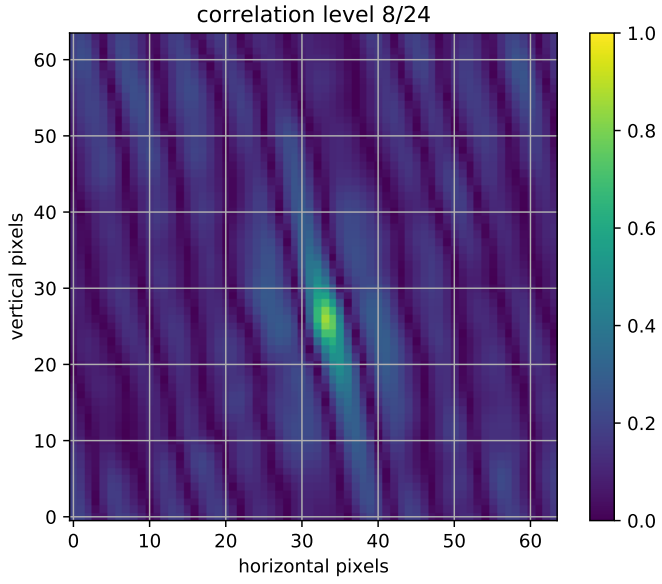

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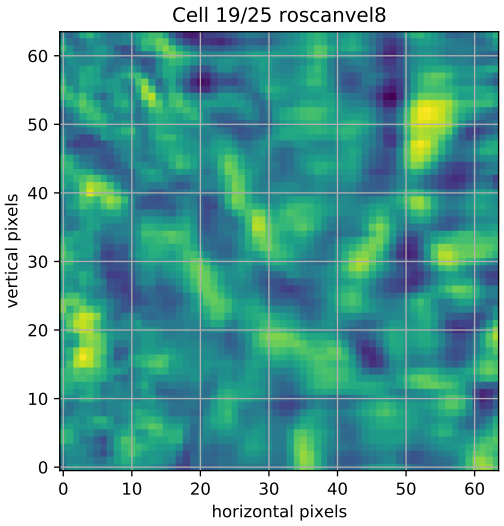
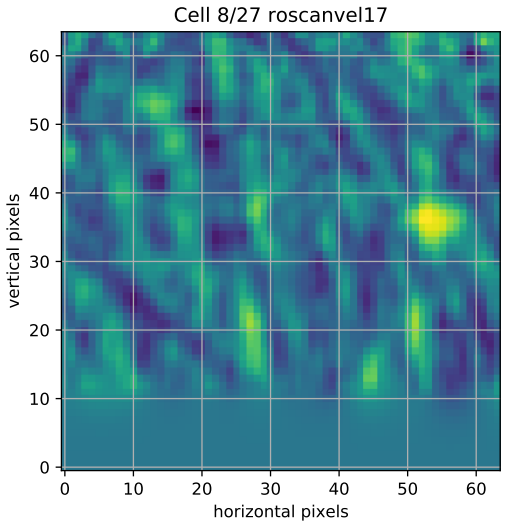
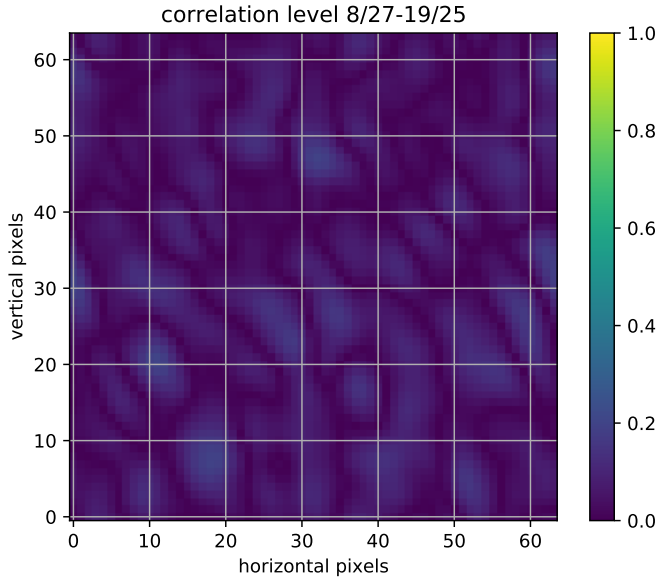
- Enclosure [**d**]

  based on some confidence level

Non-correlation of two ground observations

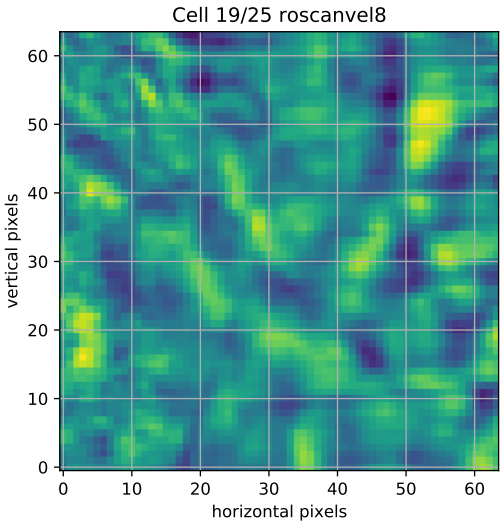


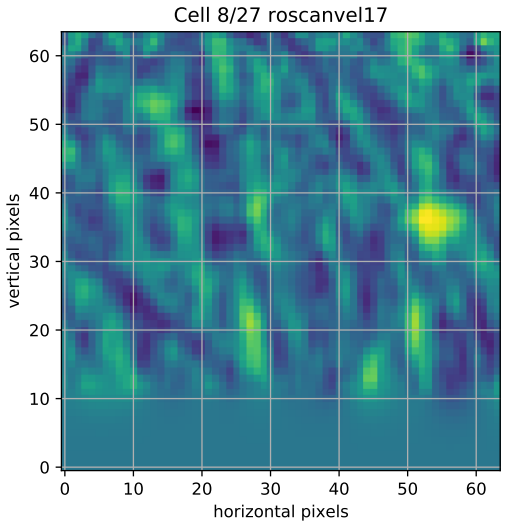


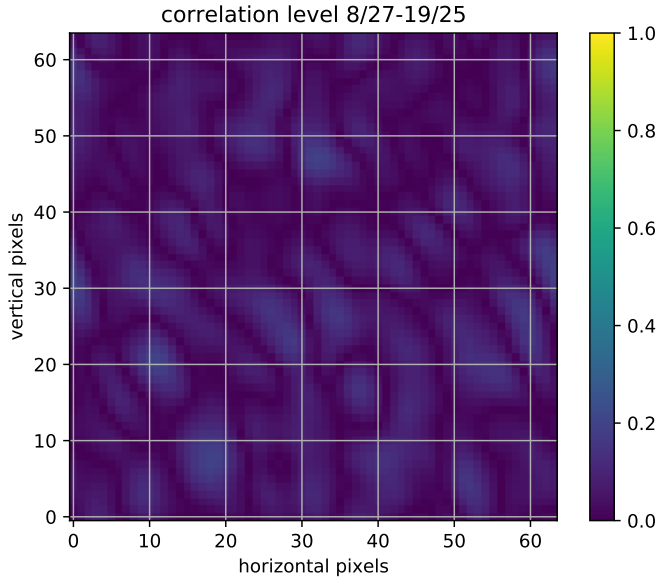


– Correlation: 0.17

Non-correlation of two ground observations







- Correlation: 0.17

- Enclosure $[\mathbf{d}] = [-\infty, \infty]^2$
  (unreliable loop closure)

Localization: constraint programming approach

State estimation associated with one loop.
One t-pair $(\tilde{t_1}, \tilde{t_2})$ is selected randomly in a verified loop-set $\mathbb{T}_i$.

- **Variables:**
  - $\mathbf{p}(\cdot)$, $\mathbf{v}(\cdot) \in [t_0, t_f] \rightarrow \mathbb{R}^2$
  - $\mathbf{d} \in \mathbb{R}^2$
  - $(\tilde{t_1}, \tilde{t_2}) \in \mathbb{R}^2$
- **Constraints:**
  - $\dot{\mathbf{p}}(t) = \mathbf{v}(t)$
  - $\mathbf{p}(\tilde{t_1}) + \mathbf{d} = \mathbf{p}(\tilde{t_2})$

Localization: constraint programming approach

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Decomposition

- **Variables:**
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  - $\mathbf{d}$, $\mathbf{q}_1$, $\mathbf{q}_2 \in \mathbb{R}^2$
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- **Constraints:**
  - $\dot{\mathbf{p}}(t) = \mathbf{v}(t)$
  - $\mathbf{q}_1 = \mathbf{p}(\tilde{t_1})$
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  - $\mathbf{d} = \mathbf{q}_2 - \mathbf{q}_1$

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- **Interval domains:**
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- **Contractors:**

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- **Interval domains:**
  - $[\mathbf{p}](\cdot)$, $[\mathbf{v}](\cdot) \in [t_0, t_f] \to \mathbb{IR}^2$
  - $[\mathbf{d}]$, $[\mathbf{q}_1]$, $[\mathbf{q}_2] \in \mathbb{IR}^2$
- **Contractors:**
  - $\mathcal{C}_{\text{deriv}}\left([\mathbf{p}](\cdot), [\mathbf{v}](\cdot)\right)$
  - $\mathcal{C}_{\text{eval}}\left(\tilde{t_1}, [\mathbf{q}_1], [\mathbf{p}](\cdot)\right)$
  - $\mathcal{C}_{\text{eval}}\left(\tilde{t_2}, [\mathbf{q}_2], [\mathbf{p}](\cdot)\right)$
  - $\mathcal{C}_{-}\left([\mathbf{q}_2], [\mathbf{q}_1], [\mathbf{d}]\right)$

Example of application

www.codac.io

Possible implementation of the SLAM algorithm:

Append to a new contractor network:

- $\mathcal{C}_{\text{deriv}}([\mathbf{p}](\cdot), [\mathbf{v}](\cdot))$

While fixpoint is not reached:

- compute/update loop-sets $\mathbb{T}$ from $[\mathbf{p}](\cdot), [\mathbf{v}](\cdot)$
- for each connected subset $\mathbb{T}_i$ of $\mathbb{T}$
  - if zero verification on $\mathbb{T}_i$:
    - select $(\tilde{t_1}, \tilde{t_2}) \in \mathbb{T}_i$
    - init $[\mathbf{q}_2] \leftarrow [-\infty, \infty]^2$, $[\mathbf{q}_1] \leftarrow [-\infty, \infty]^2$
    - compute bounded correlation $[\mathbf{d}]$ between $\mathbf{g}(\hat{\mathbf{x}}(\tilde{t_1}))$, $\mathbf{g}(\hat{\mathbf{x}}(\tilde{t_2}))$

      Append to a contractor network:
    - $\mathcal{C}_{-}([\mathbf{d}], [\mathbf{q}_2], [\mathbf{q}_1])$
    - $\mathcal{C}_{\text{eval}}(\tilde{t_1}, [\mathbf{q}_1], [\mathbf{p}](\cdot), [\mathbf{v}](\cdot))$
    - $\mathcal{C}_{\text{eval}}(\tilde{t_2}, [\mathbf{q}_2], [\mathbf{p}](\cdot), [\mathbf{v}](\cdot))$

  Propagate contractions in the contractor network.

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Append to a new contractor network:

- $\mathcal{C}_{\text{deriv}}([\mathbf{p}](\cdot), [\mathbf{v}](\cdot))$

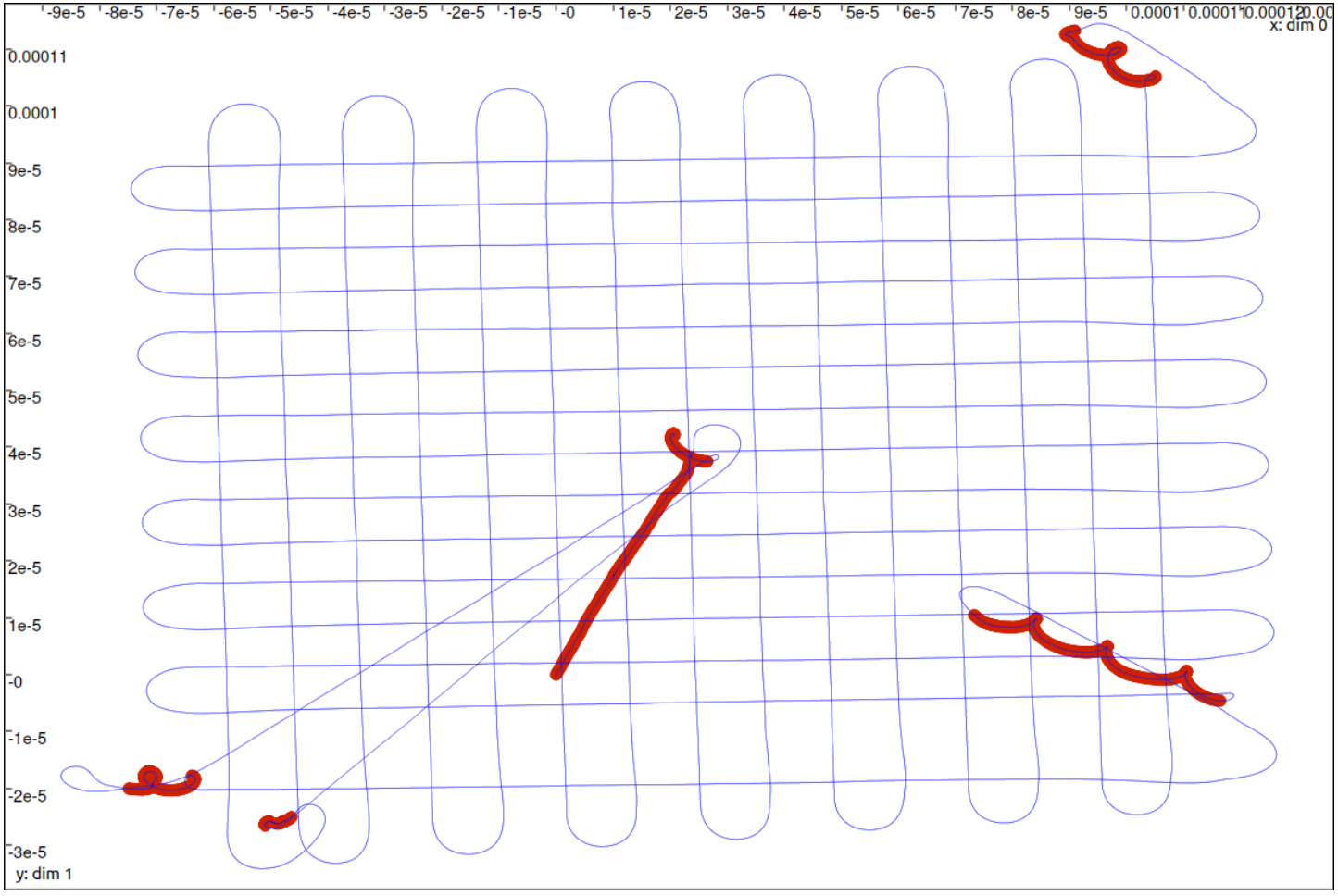
While fixpoint is not reached ($\Leftrightarrow$ while new loops are verified):

- compute/update loop-sets $\mathbb{T}$ from $[\mathbf{p}](\cdot), [\mathbf{v}](\cdot)$
- for each connected subset $\mathbb{T}_i$ of $\mathbb{T}$
  - if zero verification on $\mathbb{T}_i$:
    - select $(\tilde{t_1}, \tilde{t_2}) \in \mathbb{T}_i$
    - init $[\mathbf{q}_2] \leftarrow [-\infty, \infty]^2$, $[\mathbf{q}_1] \leftarrow [-\infty, \infty]^2$
    - compute bounded correlation $[\mathbf{d}]$ between $\mathbf{g}(\hat{\mathbf{x}}(\tilde{t_1}))$, $\mathbf{g}(\hat{\mathbf{x}}(\tilde{t_2}))$

      Append to a contractor network:
    - $\mathcal{C}_{-}([\mathbf{d}], [\mathbf{q}_2], [\mathbf{q}_1])$
    - $\mathcal{C}_{\text{eval}}(\tilde{t_1}, [\mathbf{q}_1], [\mathbf{p}](\cdot), [\mathbf{v}](\cdot))$
    - $\mathcal{C}_{\text{eval}}(\tilde{t_2}, [\mathbf{q}_2], [\mathbf{p}](\cdot), [\mathbf{v}](\cdot))$

Propagate contractions in the contractor network.

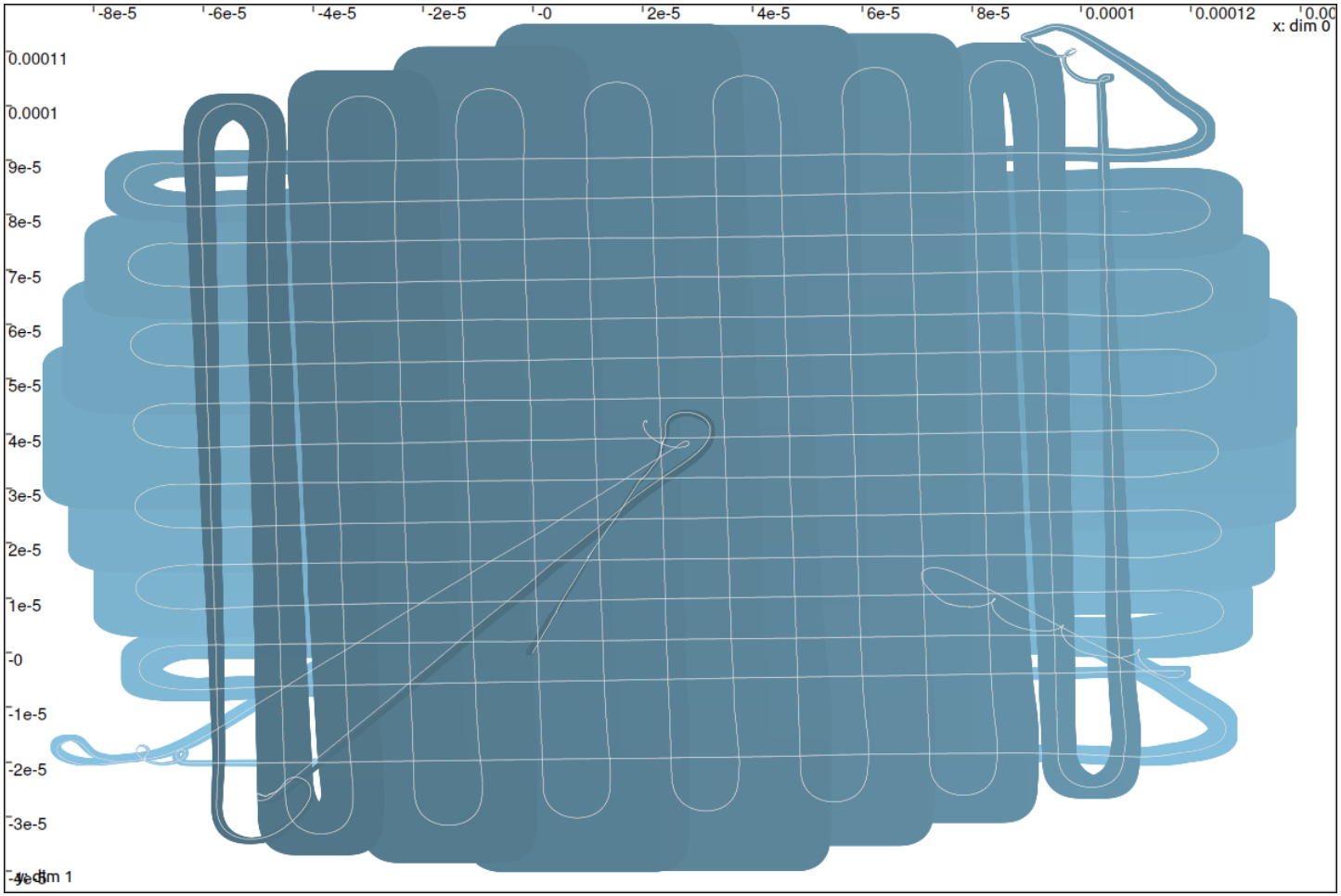
Experiment in the "Rade de Brest" (Boustrophédon)



Multi-boustrophedon pattern (actual trajectory performed by Daurade). Red points are surface positions.

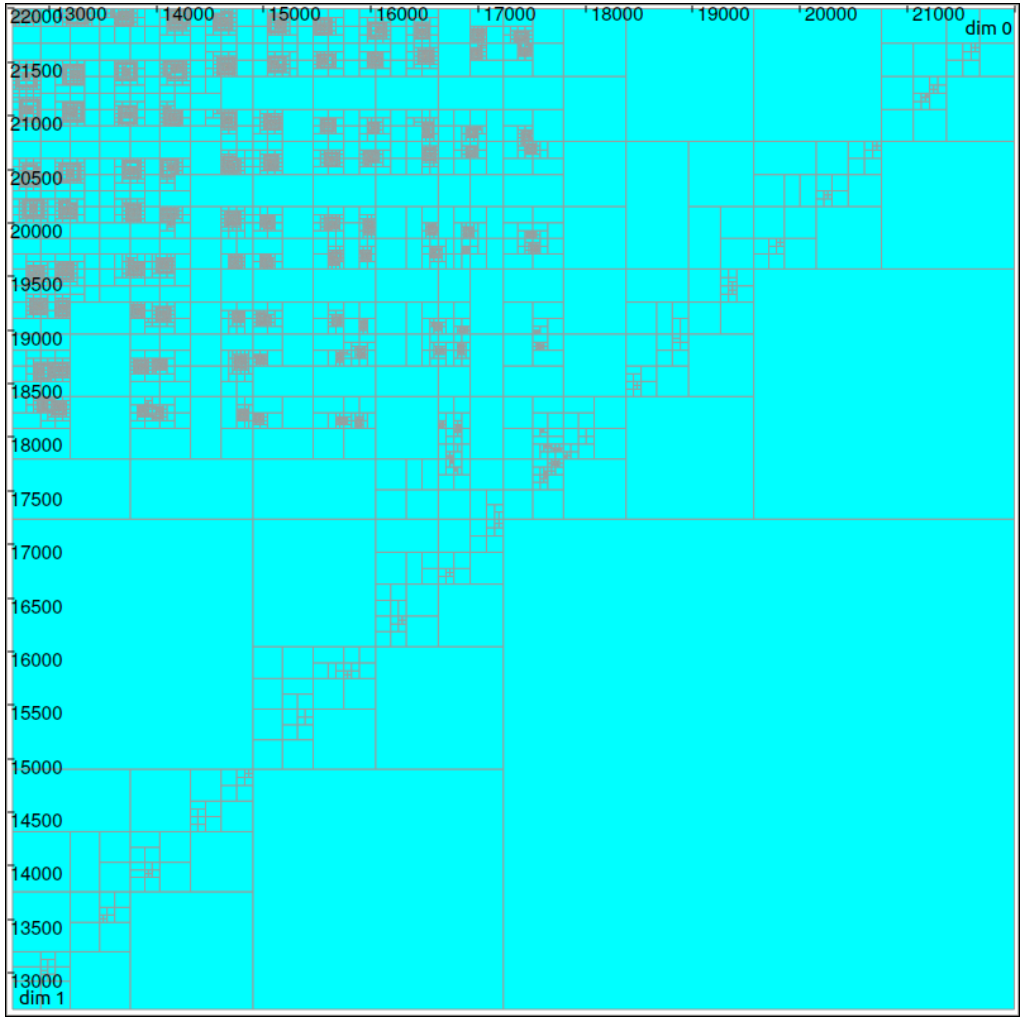
Deadreckoning

Enclosure of trajectories without external measurements:



Initial tube $[\mathbf{p}](\cdot)$ *before SLAM (inertial/DVL odometry + GNSS fixes).*

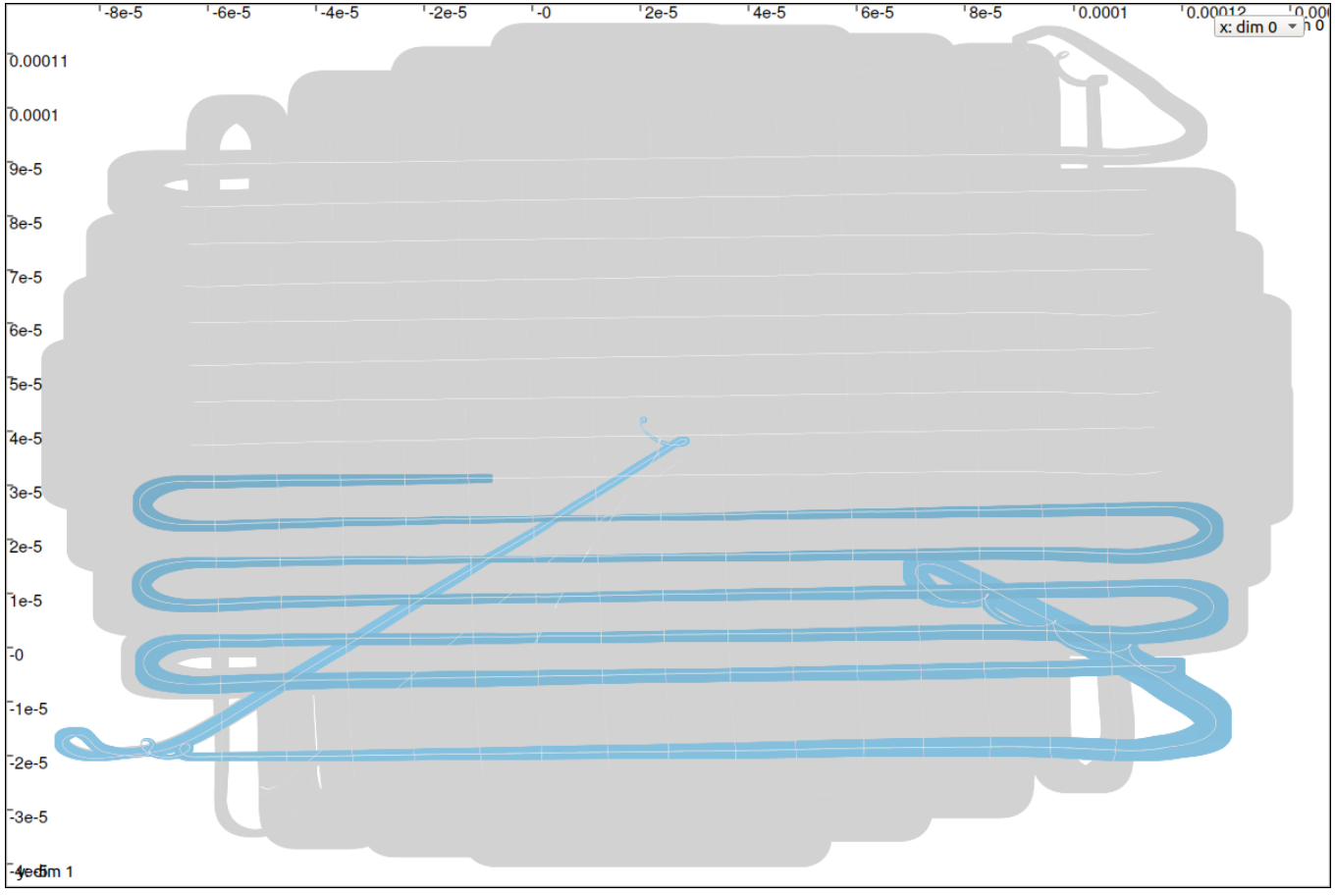
t-plane of the mission (loops summary)



Computed t-plane $[0, t_{\max}]^2$ – hundreds of loops are detected.

SLAM results

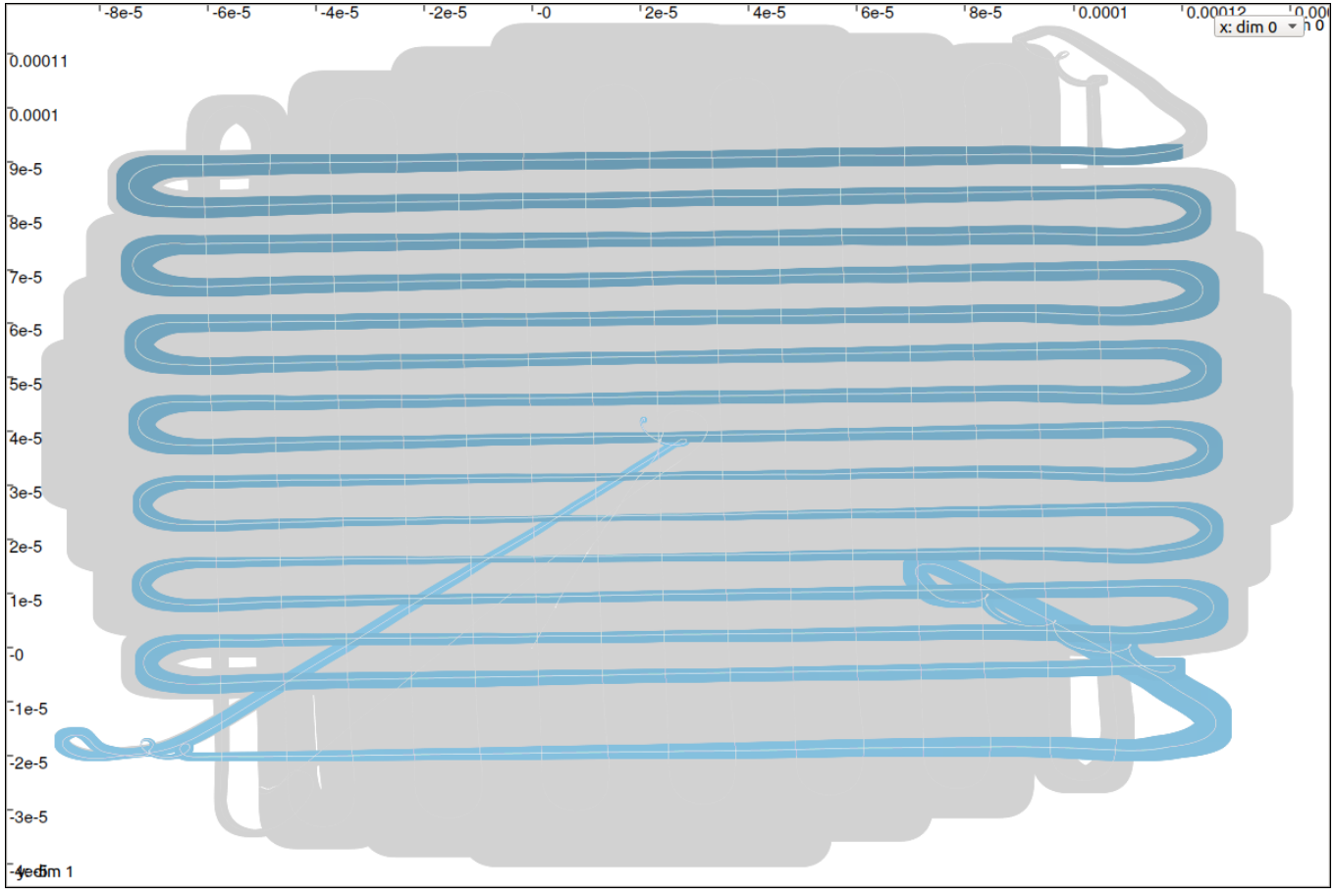
Contracted enclosure of the feasible trajectories,
with external measurements (seafloor perceptions):



Final tube $[\mathbf{p}](\cdot)$ *after SLAM.*

SLAM results

Contracted enclosure of the feasible trajectories,
with external measurements (seafloor perceptions):



Final tube $[\mathbf{p}](\cdot)$ *after SLAM.*

SLAM results

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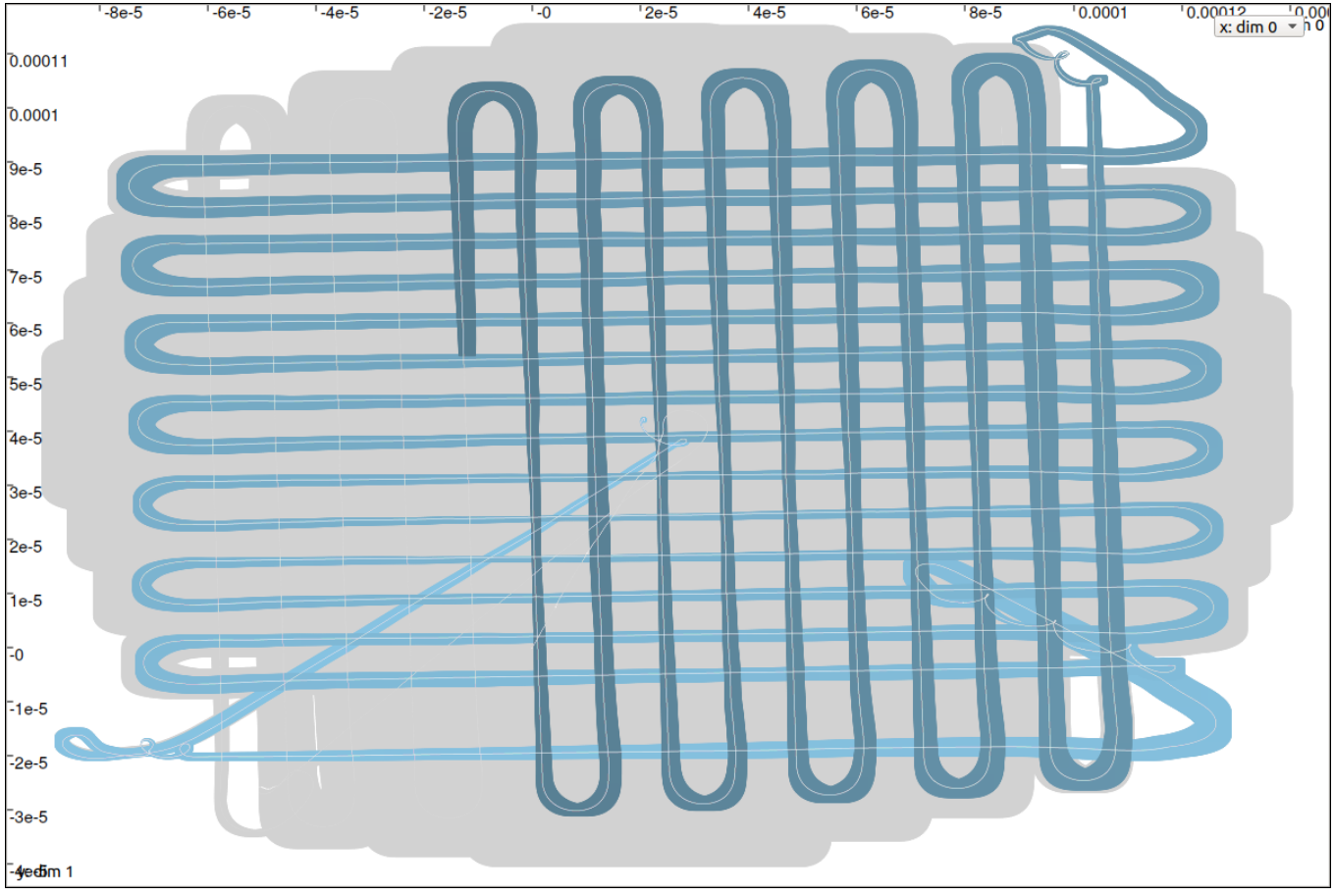

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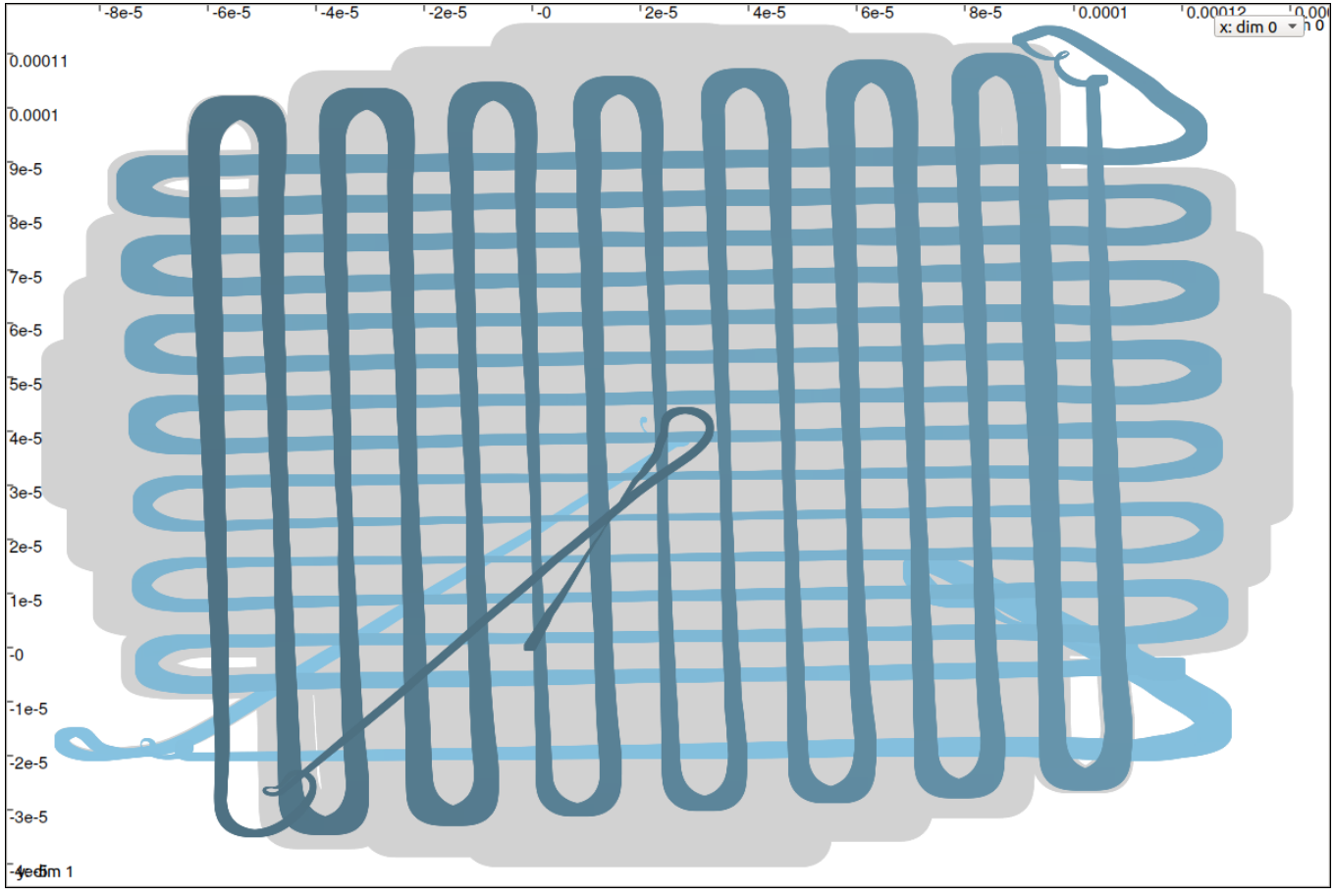
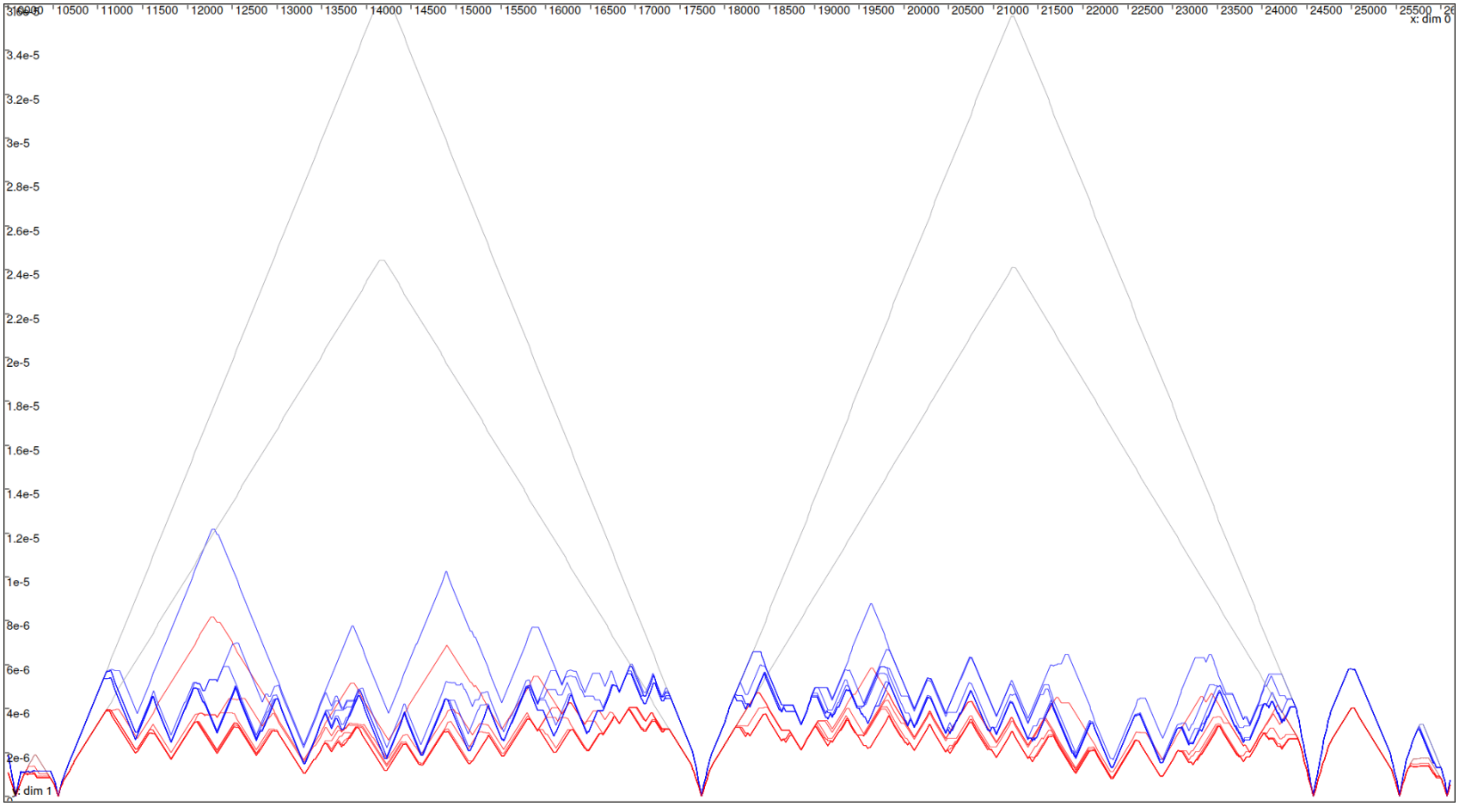


Final tube $[\mathbf{p}](\cdot)$ *after SLAM.*

SLAM results

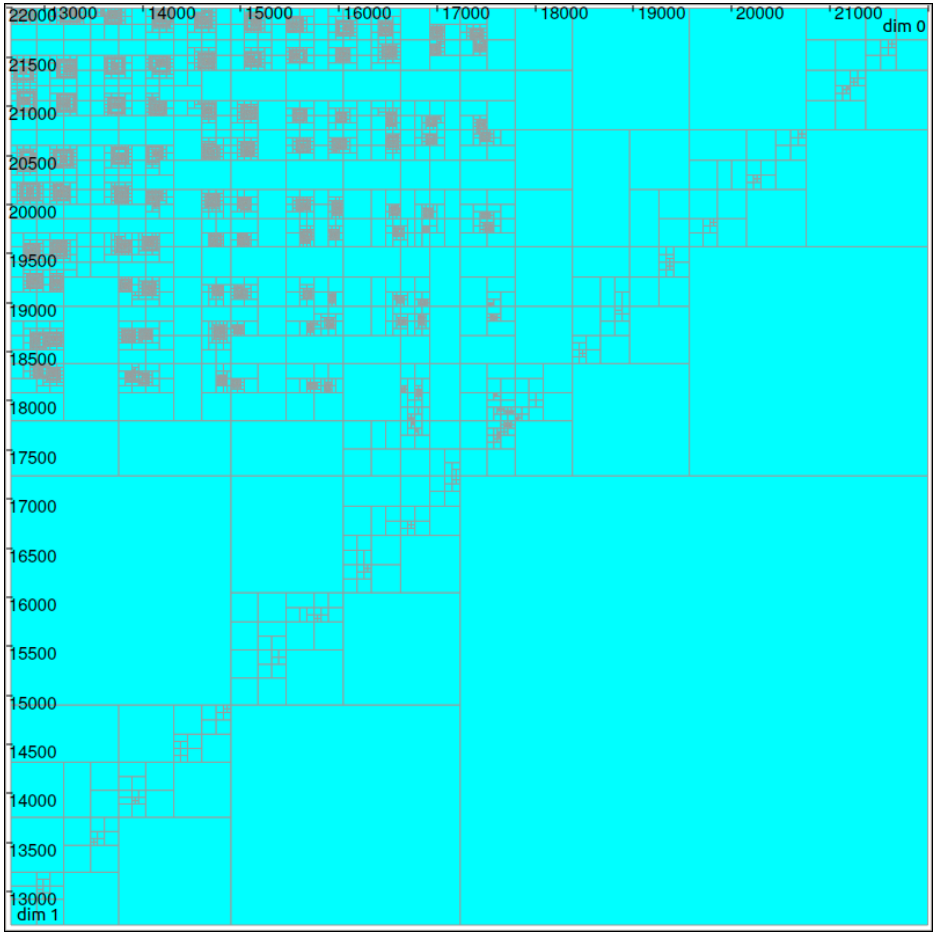


Tubes thickness along time: $\mathrm{width}([\mathbf{p}](t))$.

- *in gray: tubes without SLAM (deadreckoning only = significant drift)*
- *in blue/red: with SLAM in* p_1/p_2

t-plane of the mission (iterative computations)

The associated t-plane is iteratively contracted with respect to improvements on $[\mathbf{p}](t)$:



Computed t-plane $[0, t_{\max}]^2$ *– revealing old loop sets boundaries.*

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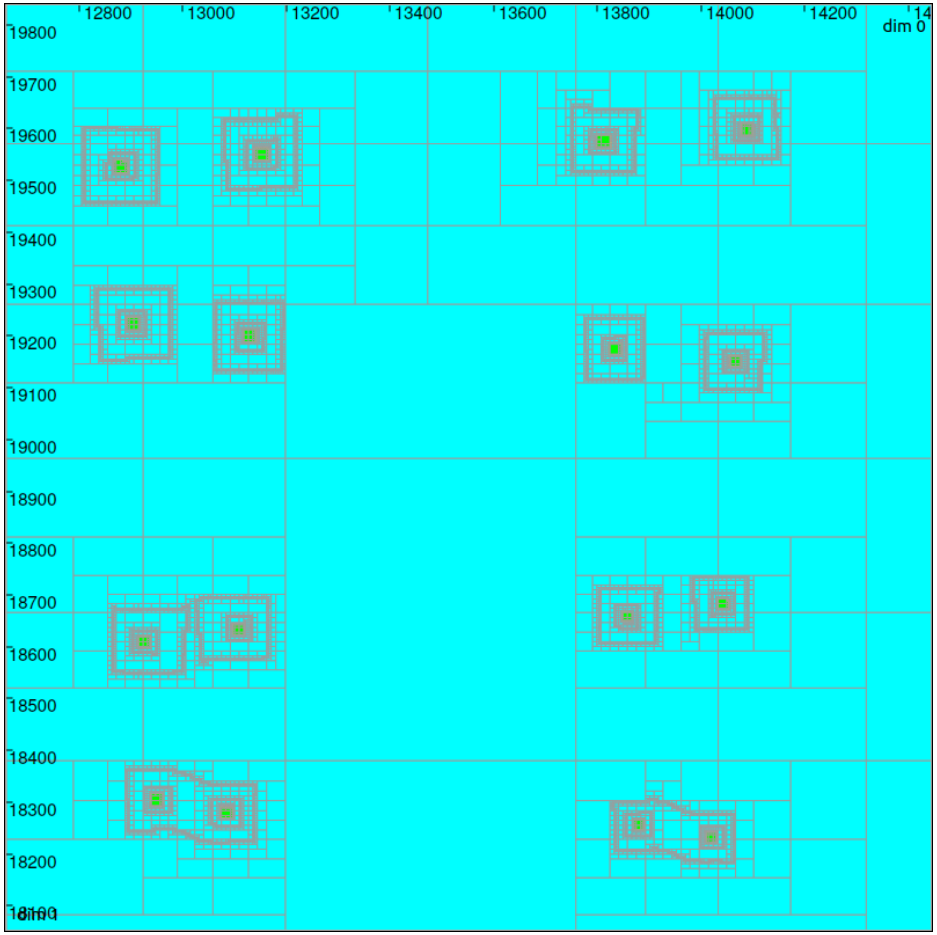

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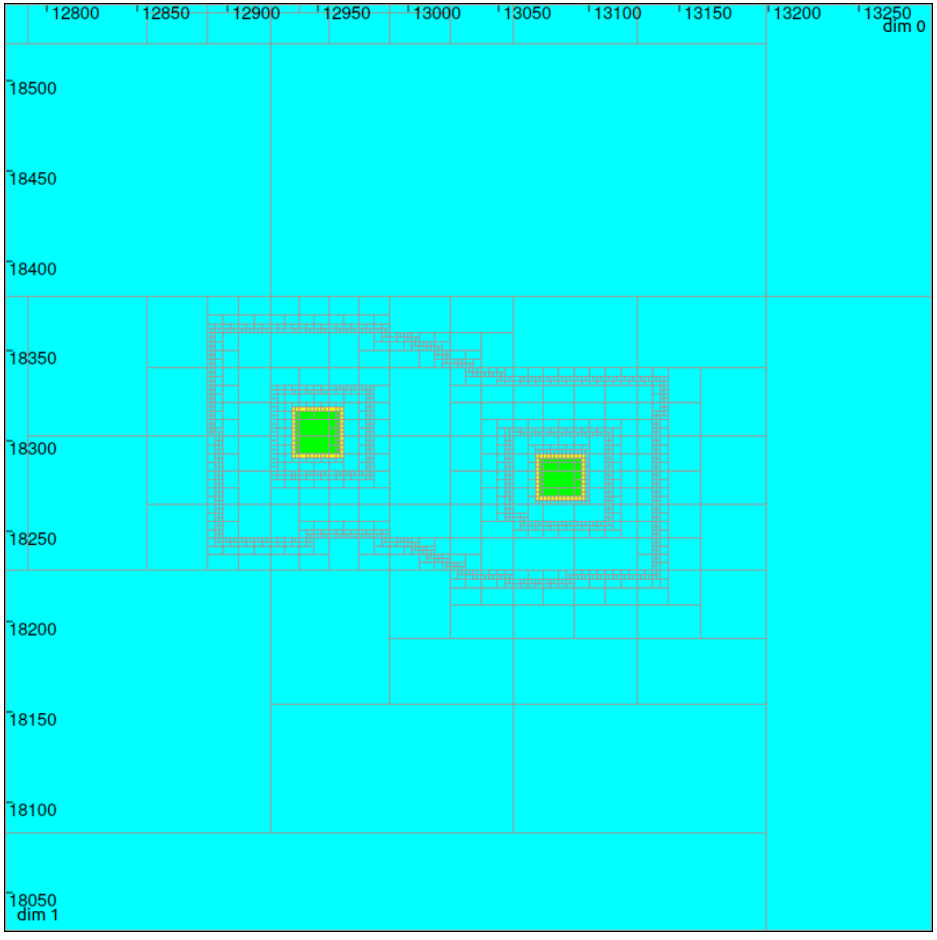

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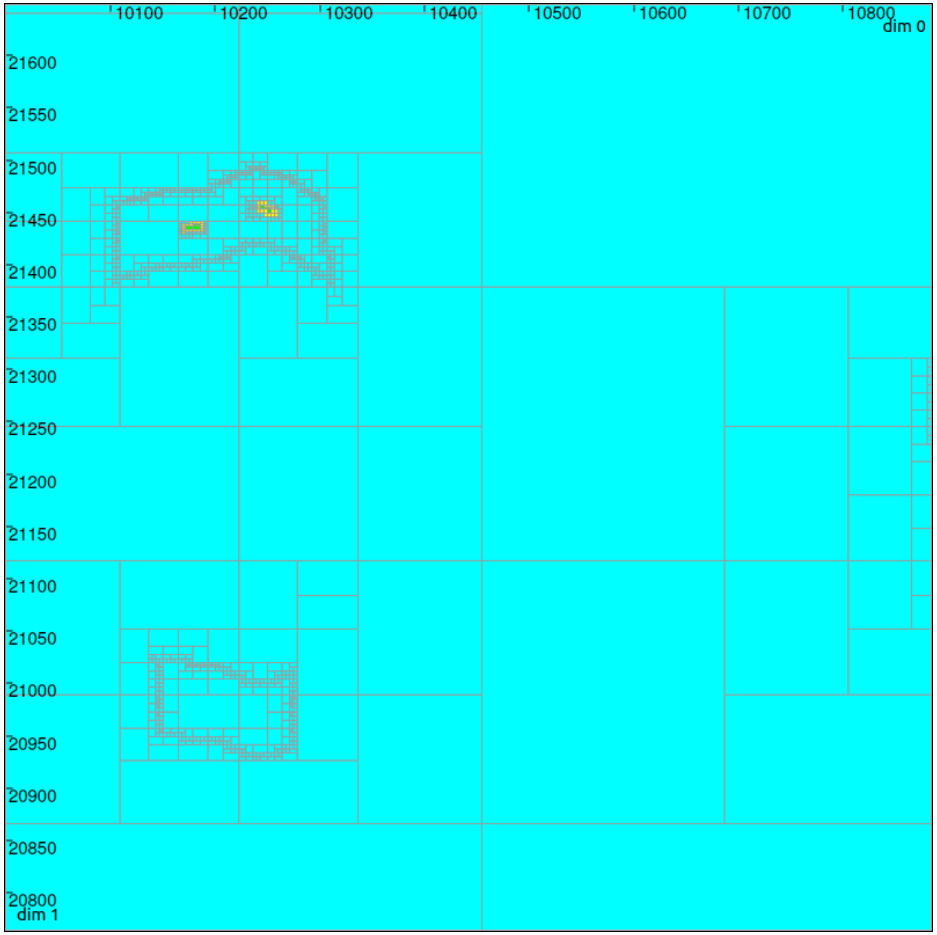

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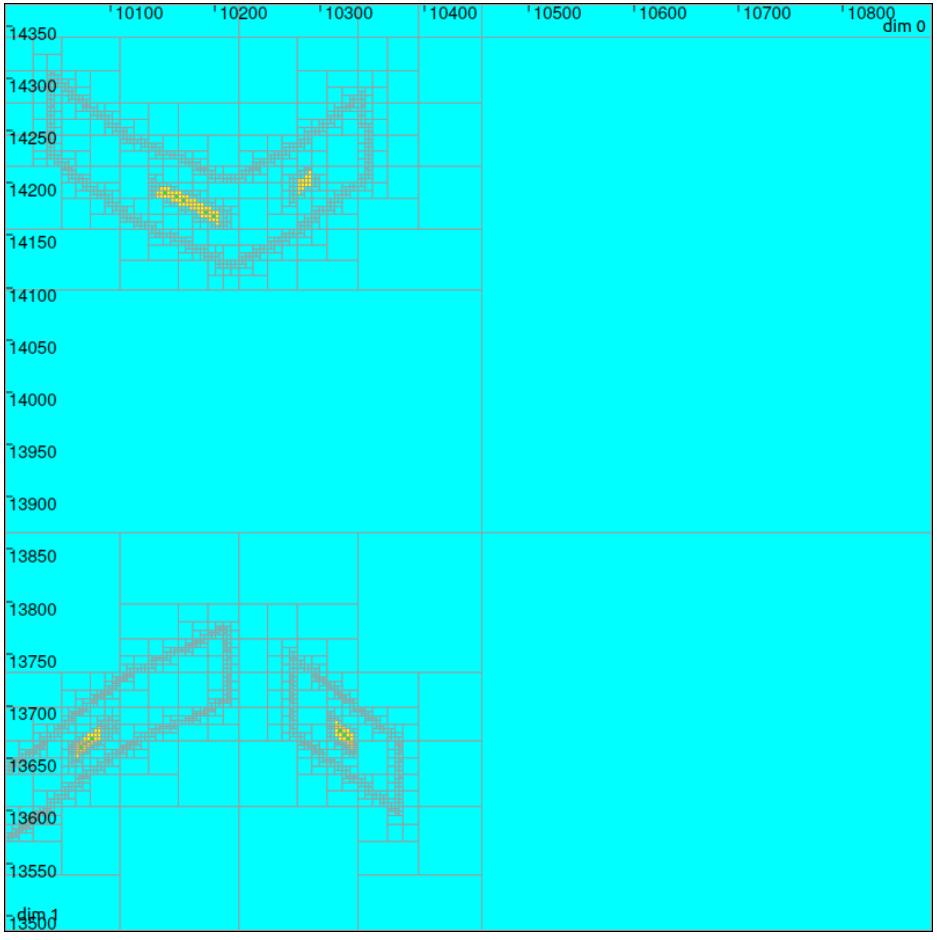

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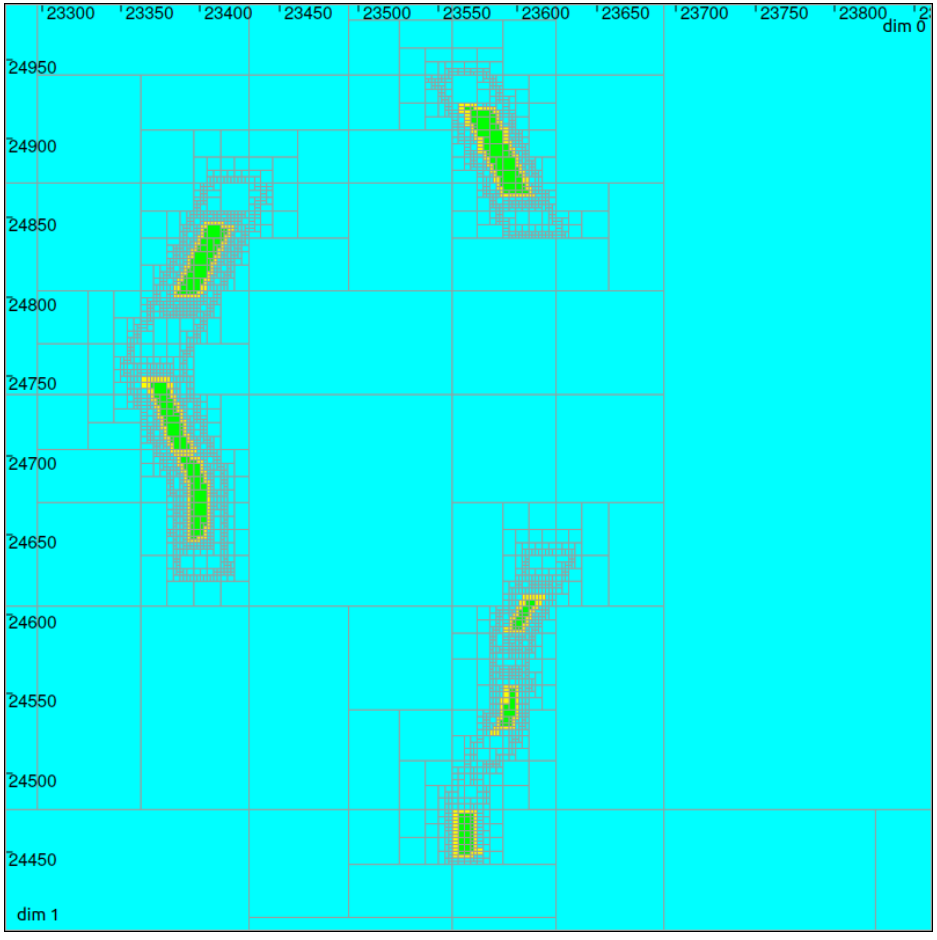

Computed t-plane $[0, t_{\max}]^2$ *– revealing old loop sets boundaries.*

Conclusion and perspectives

Assets of this approach:

- analyzing ground textures is efficient even over flat seabeds, or during strong tidal changes
- but **requires reliable localization**, whatever:
  - the ground ambiguities/similarities
  - the uncertainties on loops

Future work:

- online SLAM!

Thank you!