

Towards polar underwater exploration

Matinée "Maths pour l'Océan"
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Project to be submitted to the PEPR Maths-Vives program.

Set-membership underwater localization

Experimental missions with the AUVs

- Example: Daurade, Autonomous Underwater Vehicle (AUV)
- weight: 1010kg – length: 5m – max depth: 300m



Special thanks to DGA-TN Brest (formerly GESMA)

Experimental missions with the Daurade AUV

Video

Problem statement

Let us consider the following non-linear system:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)), \tag{1a}$$

$$y_i = g(\mathbf{x}(t_i)), \tag{1b}$$

- no prior knowledge about the states $\mathbf{x}(t) \in \mathbb{R}^n$
- but a discrete set of non-linear state observations $y_i \in \mathbb{R}$

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The problem is difficult:

- non-linearities in $\mathbf{f}$, g
- uncertainties on $\mathbf{u}(\cdot)$, y_i, t_i, ...
- no initial condition

  $\implies \mathbf{x}(t) \in [-\infty, \infty]^n$

  $\implies$ no linearization point

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  $\Longrightarrow$ no linearization point

$\Rightarrow$ can be easily dealt with interval methods, but not in any cases

Range-only localization: dynamic case

Video

Domains (wrappers).

`http://codac.io/v2`

- for reals $x \in \mathbb{R}$, $\mathbf{x} \in \mathbb{R}^n$: intervals $[x]$ and boxes $[\mathbf{x}]$
- for trajectories $x(\cdot) : \mathbb{R} \to \mathbb{R}$: tubes $[x](\cdot)$

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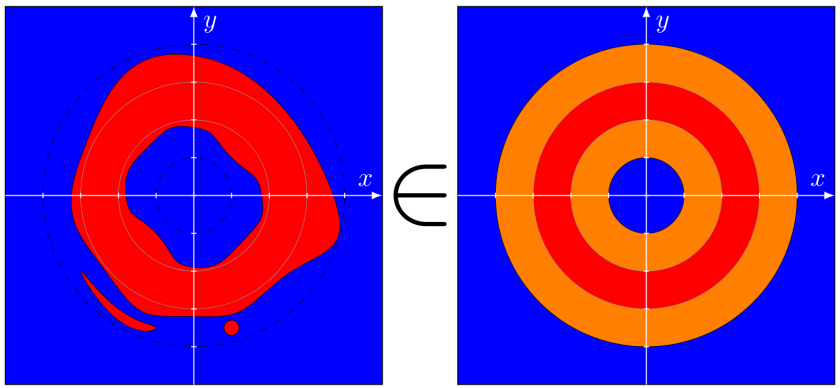


Illustration of a thickset (right-hand side) for enclosing an uncertain red set (left-hand side)

■ Thick set inversion
Desrochers, Jaulin. *Artificial Intelligence. Volume 249, Issue C, Pages 1-18*, 2017

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- *etc.*

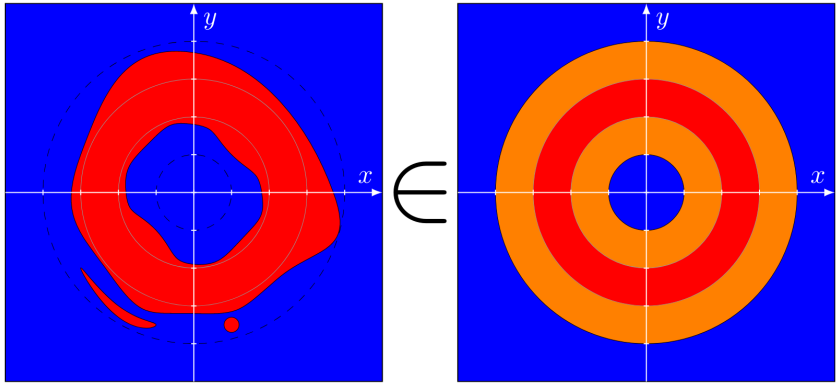


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Constraint Programming for continuous and differential variables:

Several types of **domains**:

- `Interval` (mainly from the GAOL library)
- `IntervalVector`, `IntervalMatrix` (using Eigen templates)
- `SlicedTube<T>...`, `Slice<T>...`
- `Ellipsoid`, `Paving`, `Thickset`, ...

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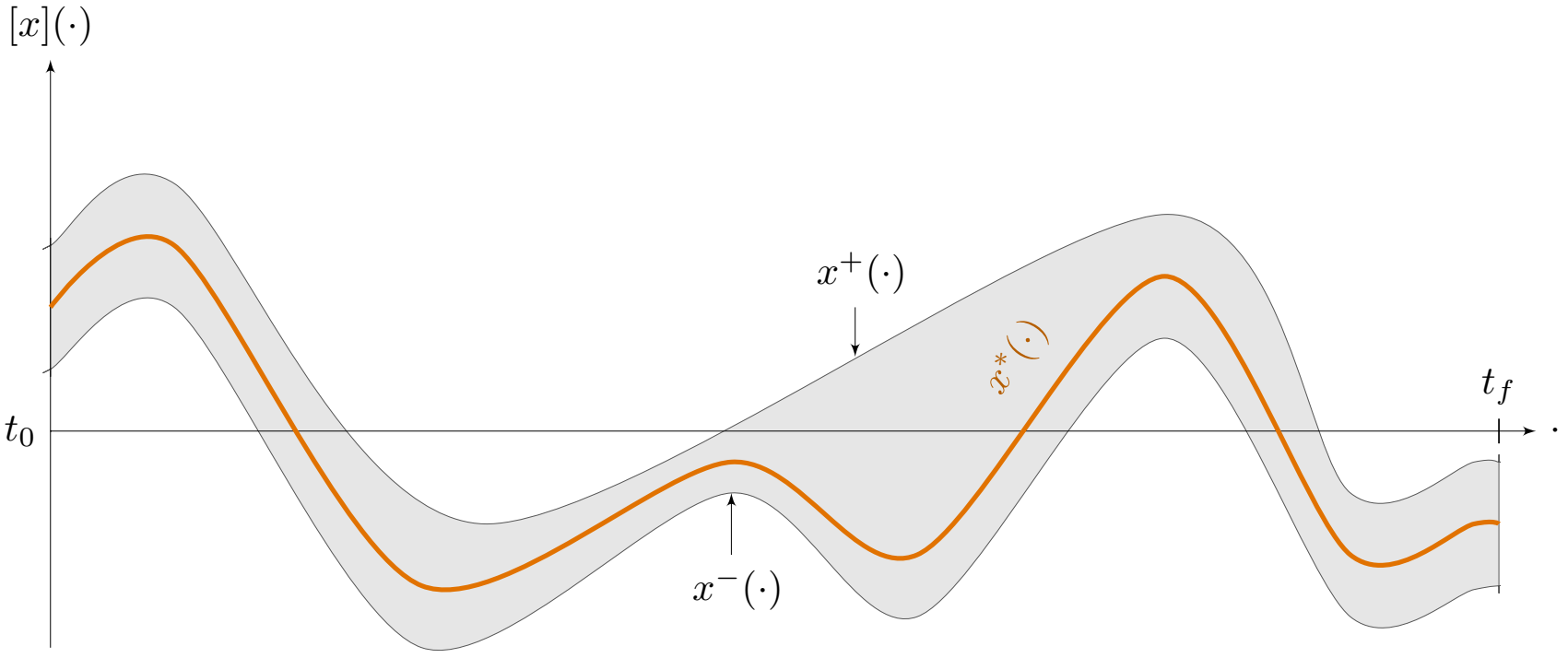
Contractors for various constraints:

- non-linear constraints $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ (involving centered form)
- geometric constraints: distance, polar equation, no-cross, ...
- differential equations: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$, $\int\int$
- time uncertainties: $\mathbf{y} = \mathbf{x}(t), \quad \text{with } t \in [t]$
- delays: $x(t) = y(t - \tau)$
- ...

Domains for trajectories: tubes

http://codac.io/v2

Tube $[x](\cdot)$**:** interval of trajectories $[x^-(\cdot), x^+(\cdot)]$
such that $\forall t \in \mathbb{R},\ x^-(t) \leqslant x^+(t)$



Example of scalar tube: interval of two trajectories

Contractors on tubes ($\mathcal{C}_{\text{eval}}$: a telling example)

http://codac.io/v2

Trajectory evaluation $\begin{cases} \mathbf{z} = \mathbf{y}(t) \\ \\ \\ \end{cases}$

- Reliable non-linear state estimation involving time uncertainties
Rohou, Jaulin, Mihaylova, Le Bars, Veres *Automatica*, 2018

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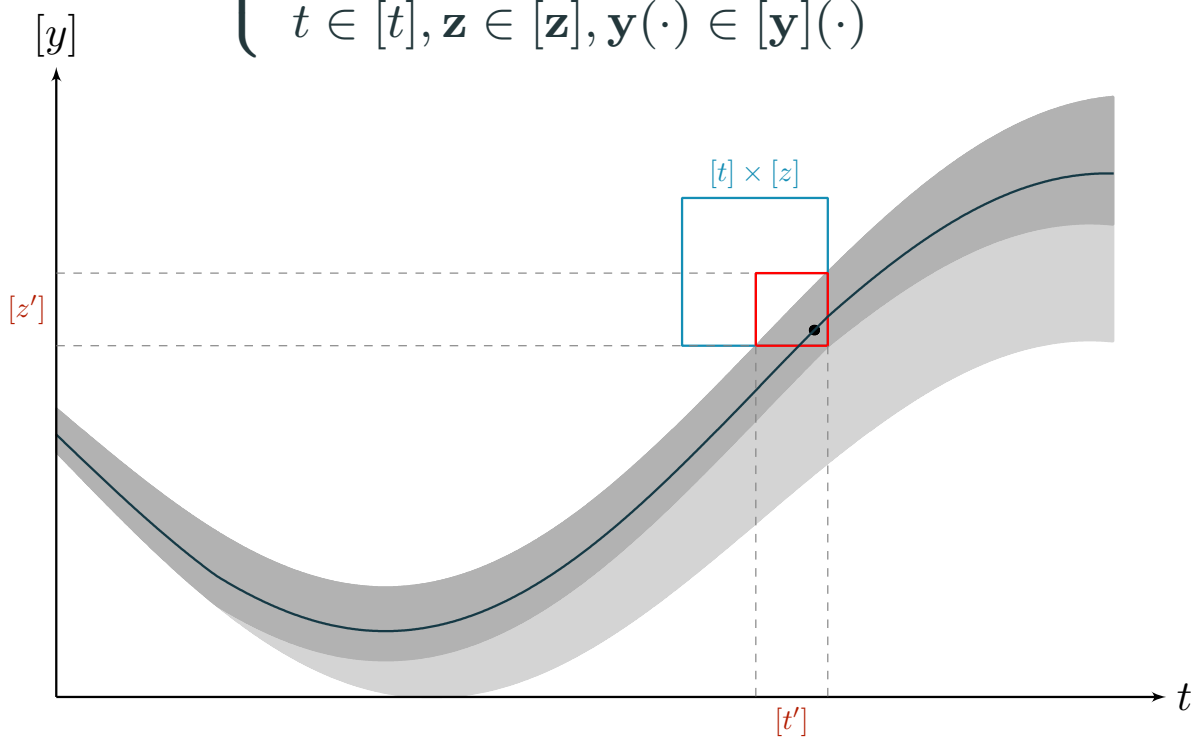
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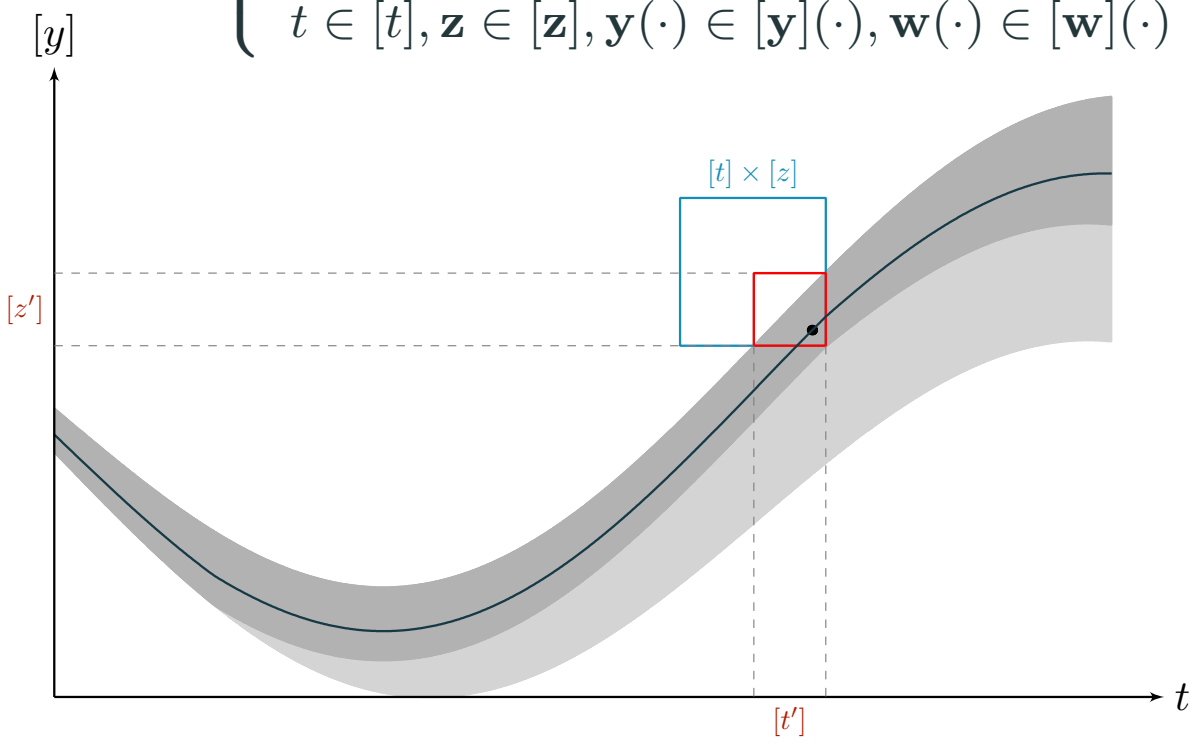


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Trajectory evaluation $\begin{cases} \mathbf{z} = \mathbf{y}(t) \\ \dot{\mathbf{y}}(\cdot) = \mathbf{w}(\cdot) \\ t \in [t], \mathbf{z} \in [\mathbf{z}], \mathbf{y}(\cdot) \in [\mathbf{y}](\cdot), \mathbf{w}(\cdot) \in [\mathbf{w}](\cdot) \end{cases}$

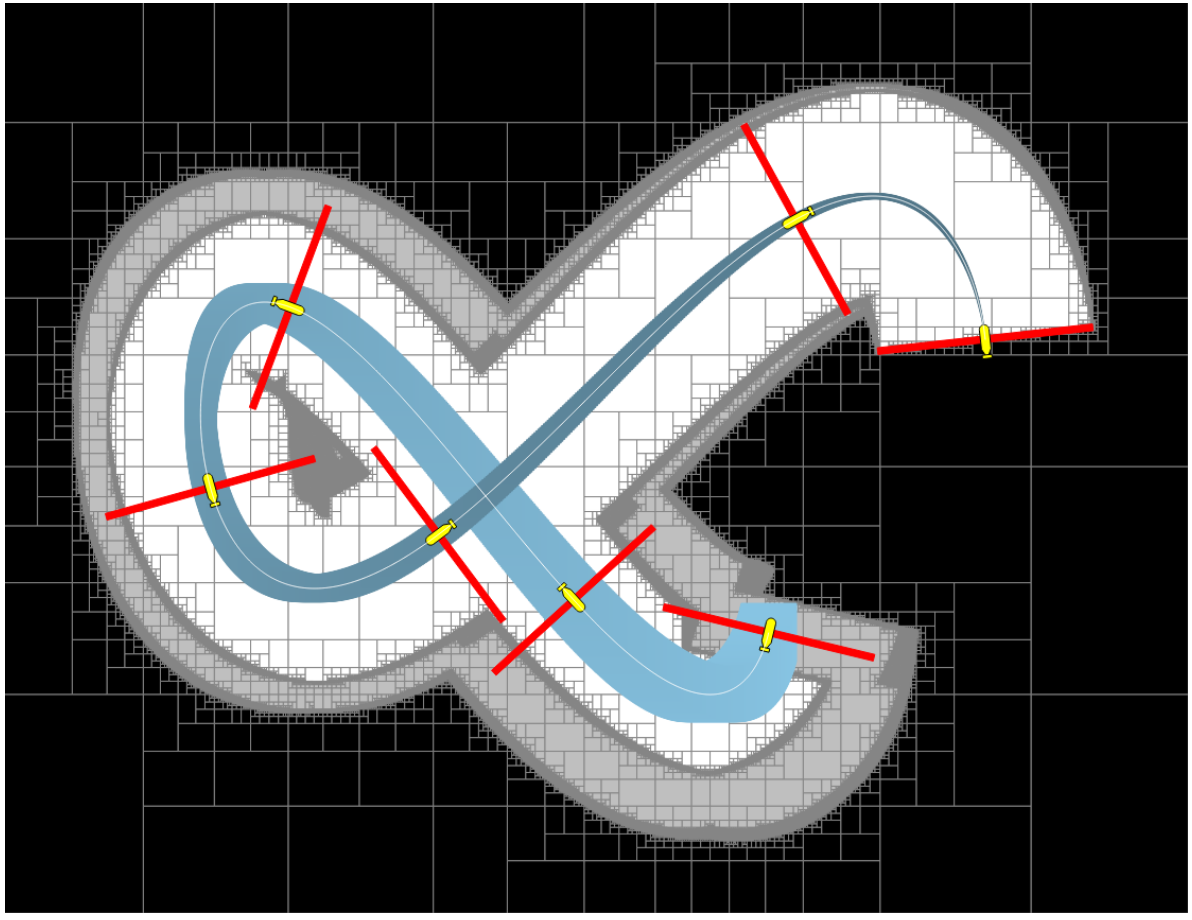


Application of the contractor: $\mathcal{C}_{\text{eval}}\big([t], [\mathbf{z}], [\mathbf{y}](\cdot), [\mathbf{w}](\cdot)\big)$

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  Rohou, Jaulin, Mihaylova, Le Bars, Veres *Automatica*, 2018

Example of mixing tubes and thicksets

`http://codac.io/v2`

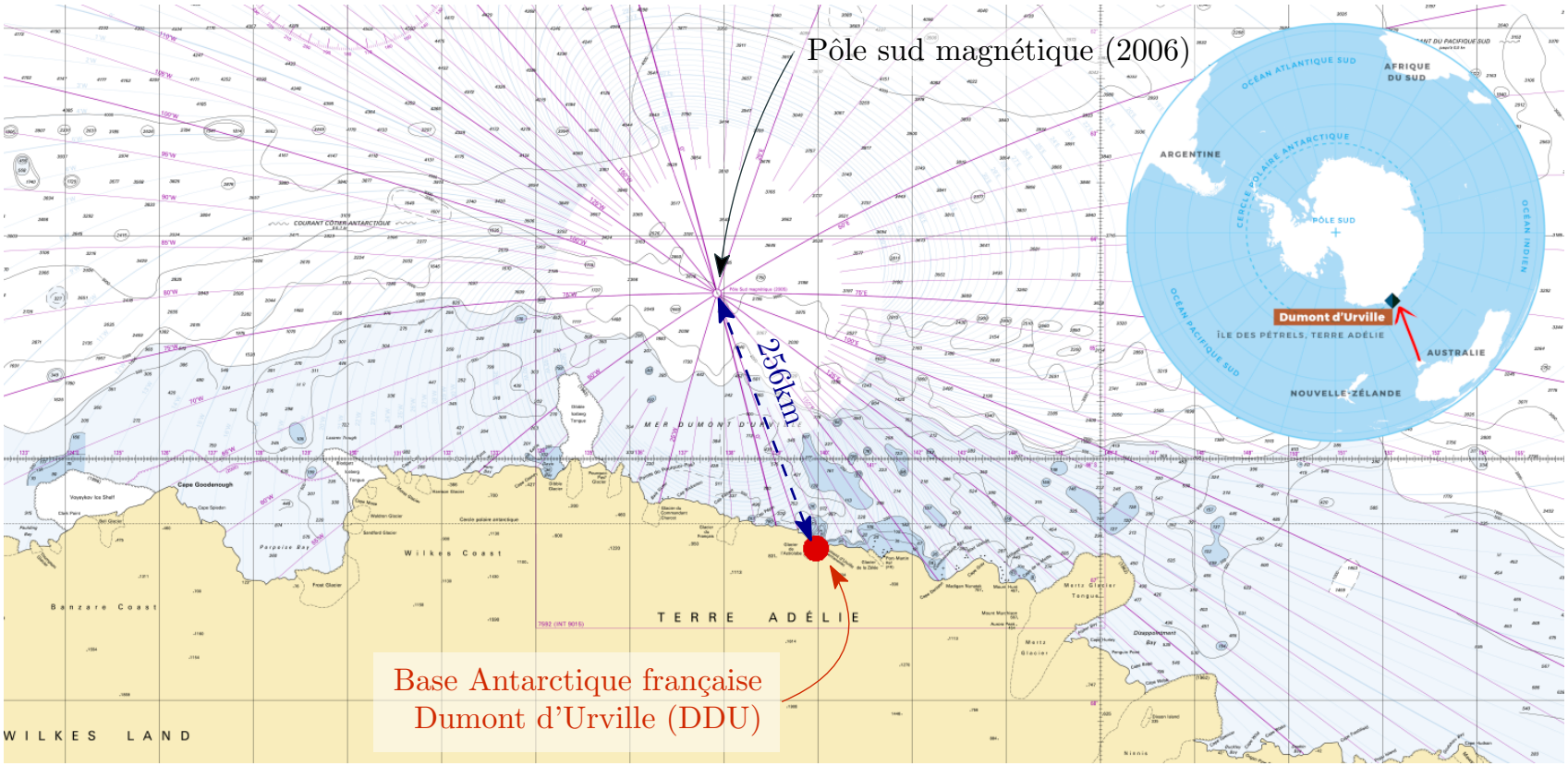


- Computing a Guaranteed Approximation of the Zone Explored by a Robot. Desrochers, Jaulin. *IEEE Trans. on Automatic Control. Volume 62, Issue 1.*, 2017

Towards underwater polar exploration

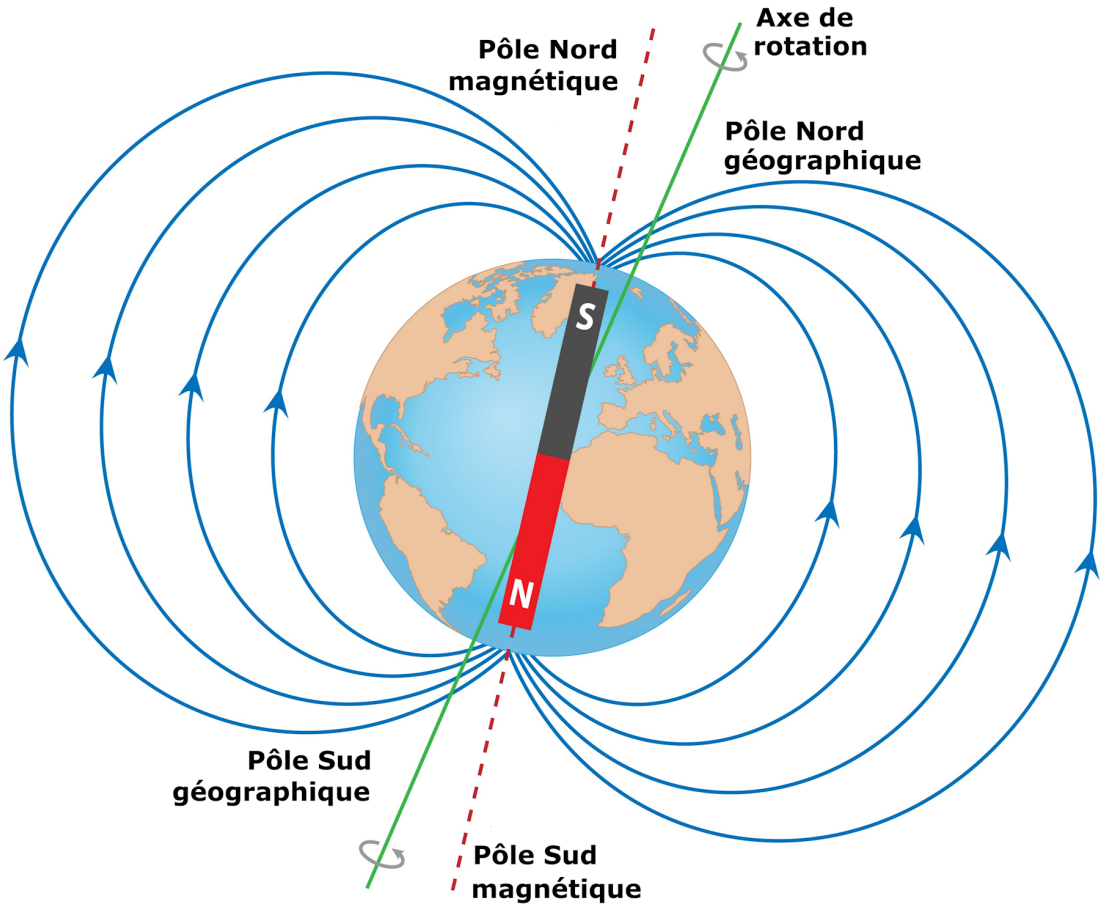
Target experimental site

French Antarctic base Dumont d'Urville (DDU), in Terre Adélie:



... only 256km from the South Magnetic Pole!

Earth's magnetic field

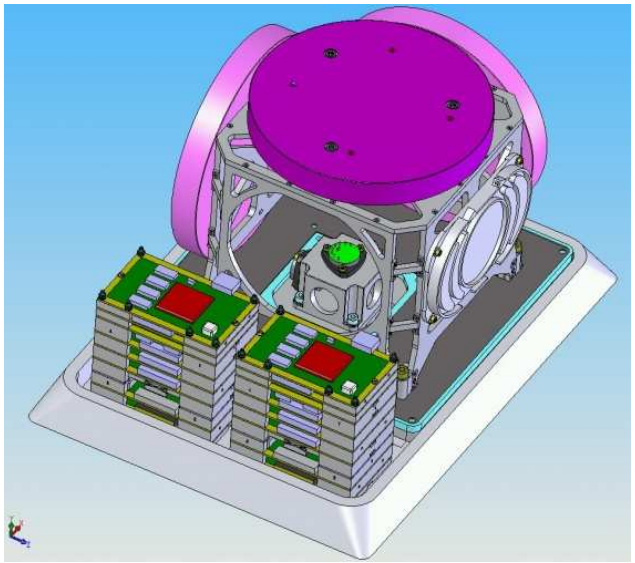


Near the pole: a magnetic compass becomes unreliable because the Earth's magnetic field is almost vertical there.

$\Longrightarrow$ its horizontal projection (the only part a compass can use to indicate heading) becomes too small.

And without a magnetic field? A gyroscopic compass?

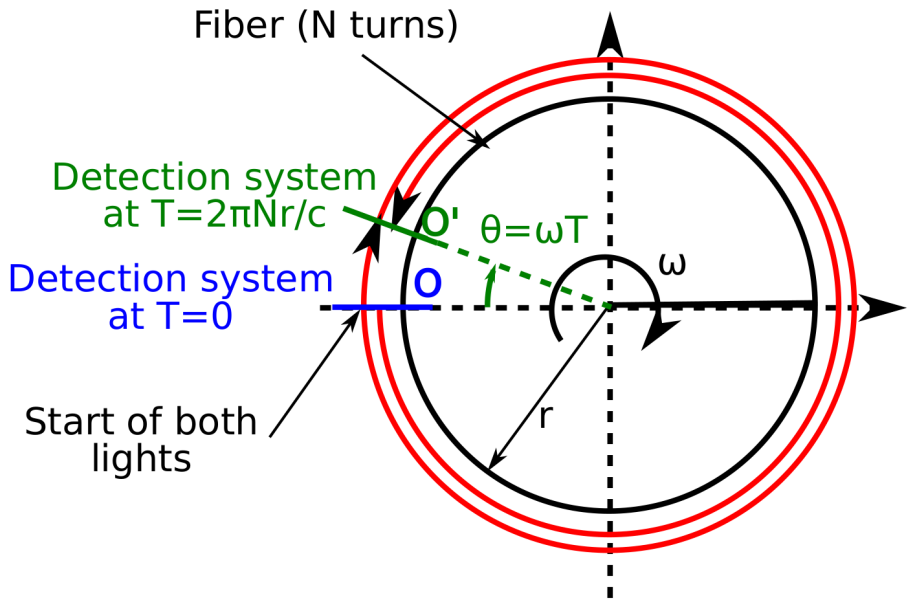
- True north estimation from Earth rotation measurement
- Independent of the Earth's magnetic field
- Typically relies on high-end FOGs, with significant cost and size constraints



- **FOG: Fiber Optic Gyroscope**
- Optical gyroscope based on the Sagnac effect

Sagnac Effect

Two coherent light beams propagate along the same optical path in opposite directions.



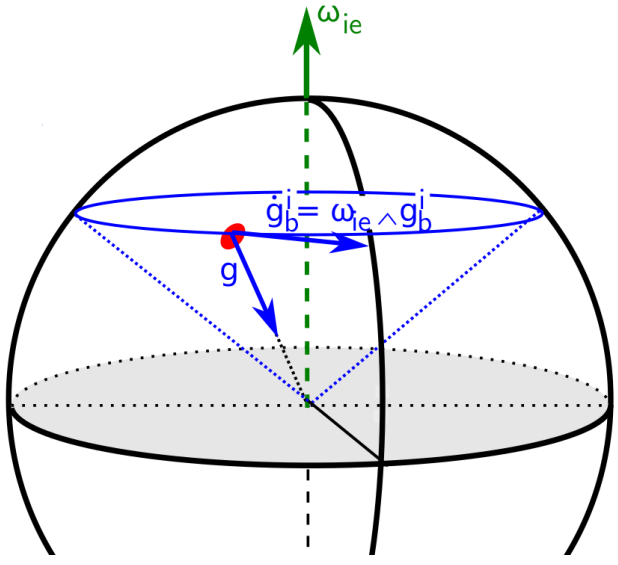
Fiber Optic Gyroscopes (FOG)

- Counter-propagating beams in a long fiber coil
- Rotation induces a measurable phase shift

Gyrocompassing

At rest:

- Gyroscope: Earth rotation rate $\boldsymbol{\omega}$
- Accelerometer: gravity vector $-\boldsymbol{g}$

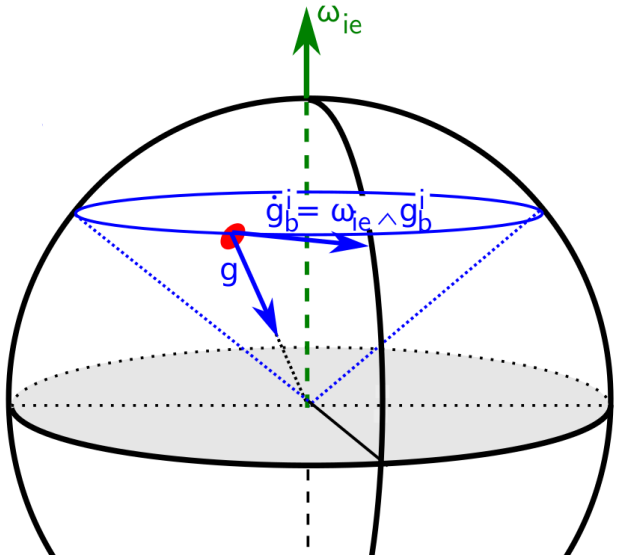


In an inertial frame, $\boldsymbol{g}$ traces a cone over 24 hours around the Earth rotation axis.

Gyrocompassing: Polar Limitation

However...

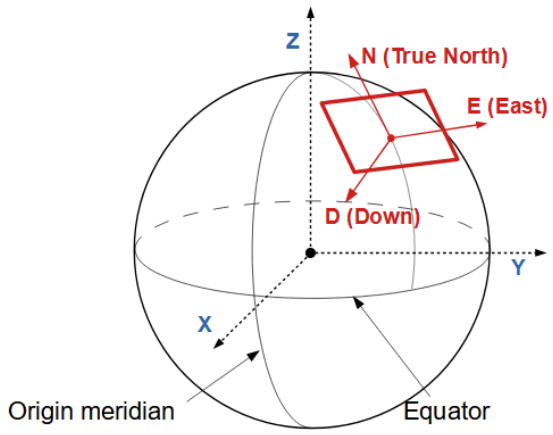
- Useful horizontal signal: $\omega \cos(\text{latitude})$
- This term decreases with latitude and vanishes at the pole
- Gyrocompassing becomes ill-conditioned at high latitudes



Polar navigation: state of the art

- No gyrocompass does not mean no gyroscopes
- Polar-adapted navigation frames are required, since the NED frame becomes ill-conditioned near the poles due to meridian convergence.
- Acoustic aiding remains a robust and accurate solution

  ... however, it is operationally expensive: deployment, calibration, recovery, and infrastructure support



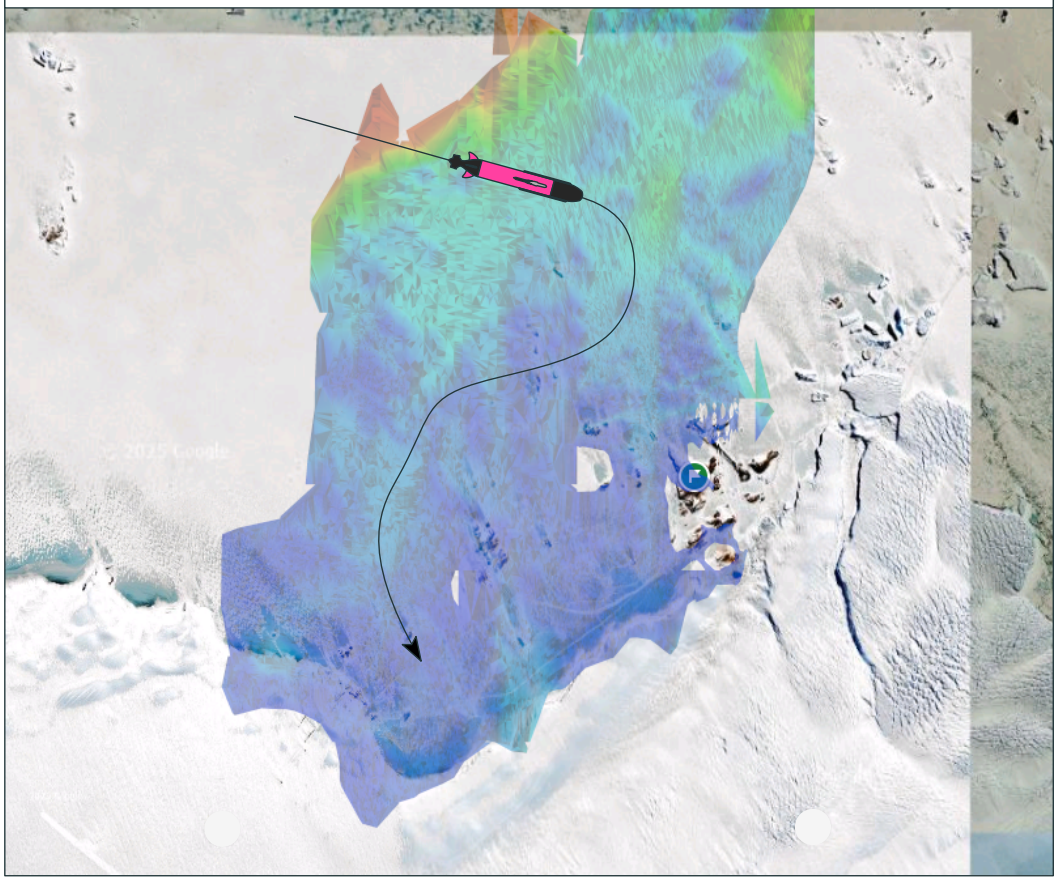
[1] NED (North–East–Down) is a local navigation frame whose axes point toward geographic north, east, and downward along the local vertical.

Solution 1:
Terrain-based navigation

Terrain-based navigation in polar regions

Polar underwater exploration: mathematical solutions to compensate for the absence of magnetic heading.

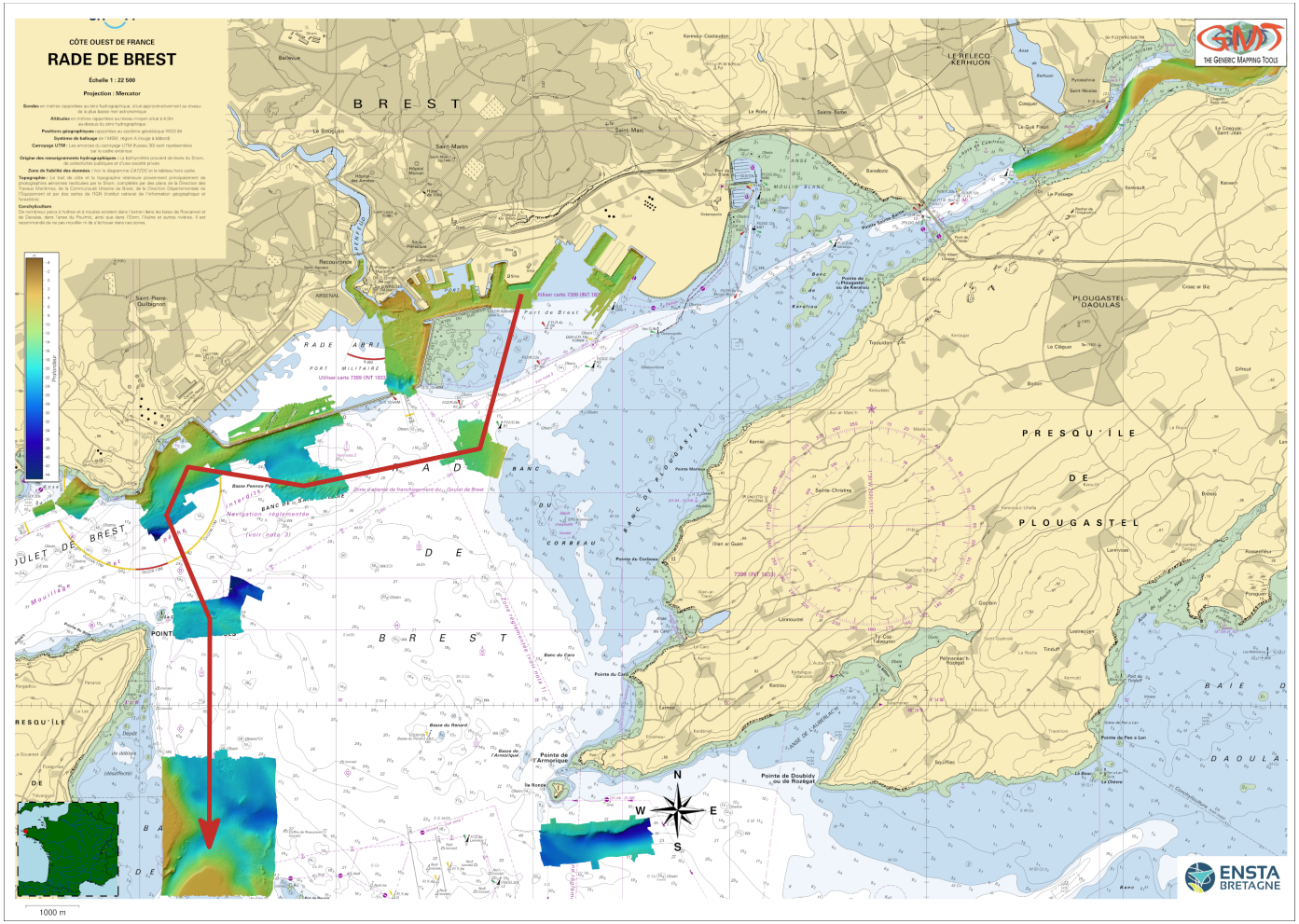
Study 1–A:
Terrain-based navigation without magn. compass



“Objectif Rade” project: crossing the rade without surfacing

10km, 10m, 1m

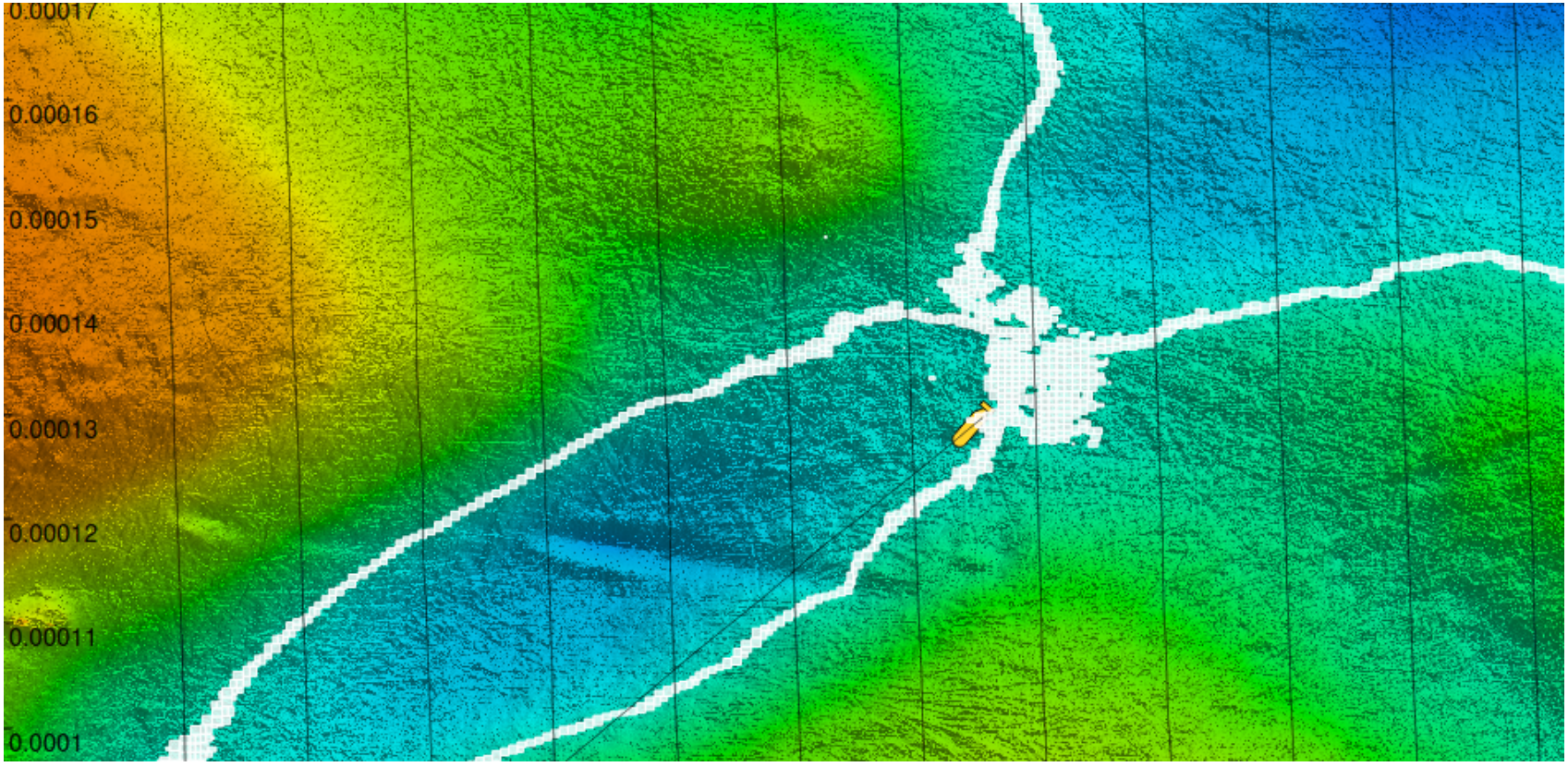
Crossing Brest harbor without surfacing using a low-cost AUV.



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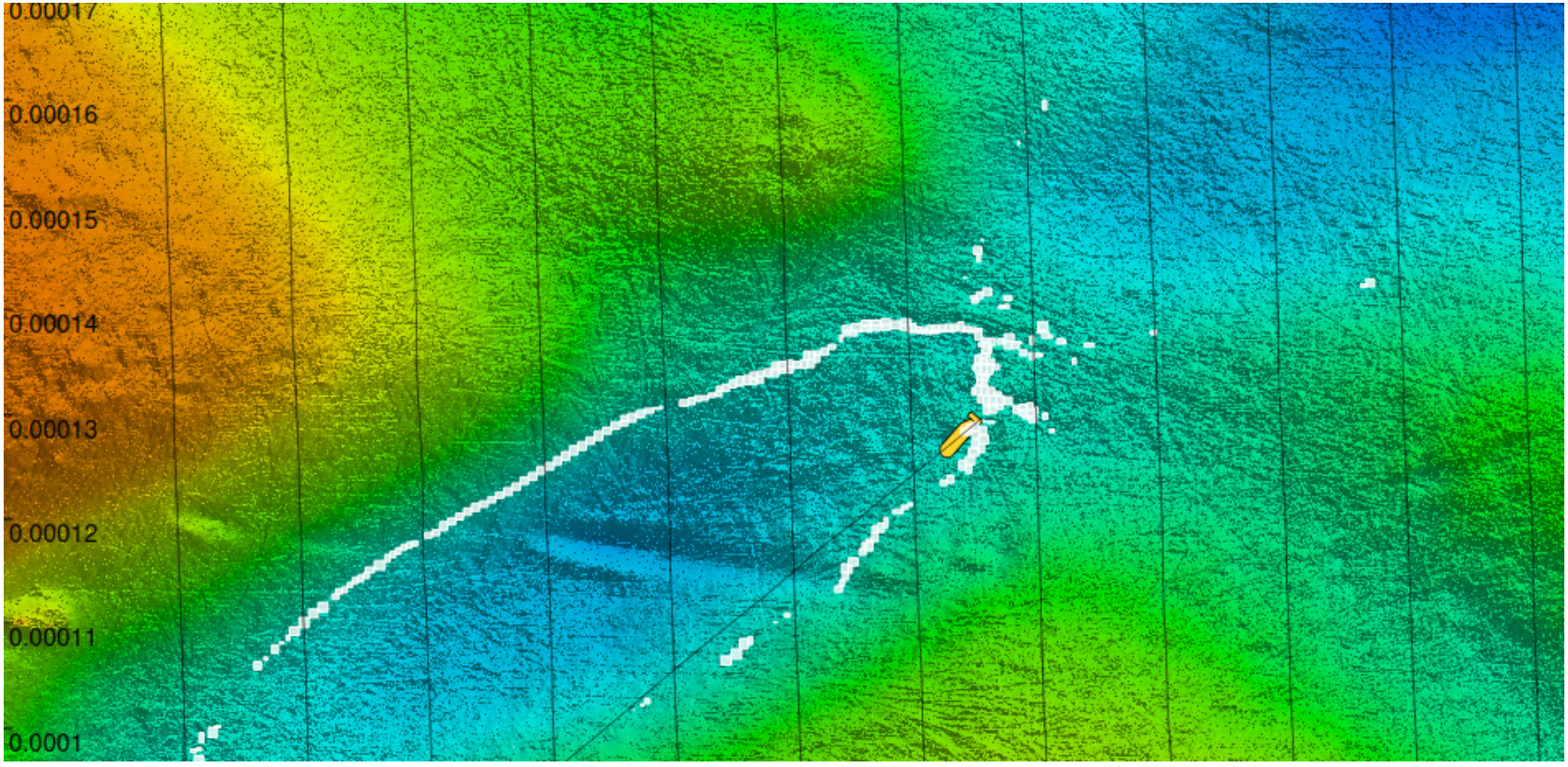
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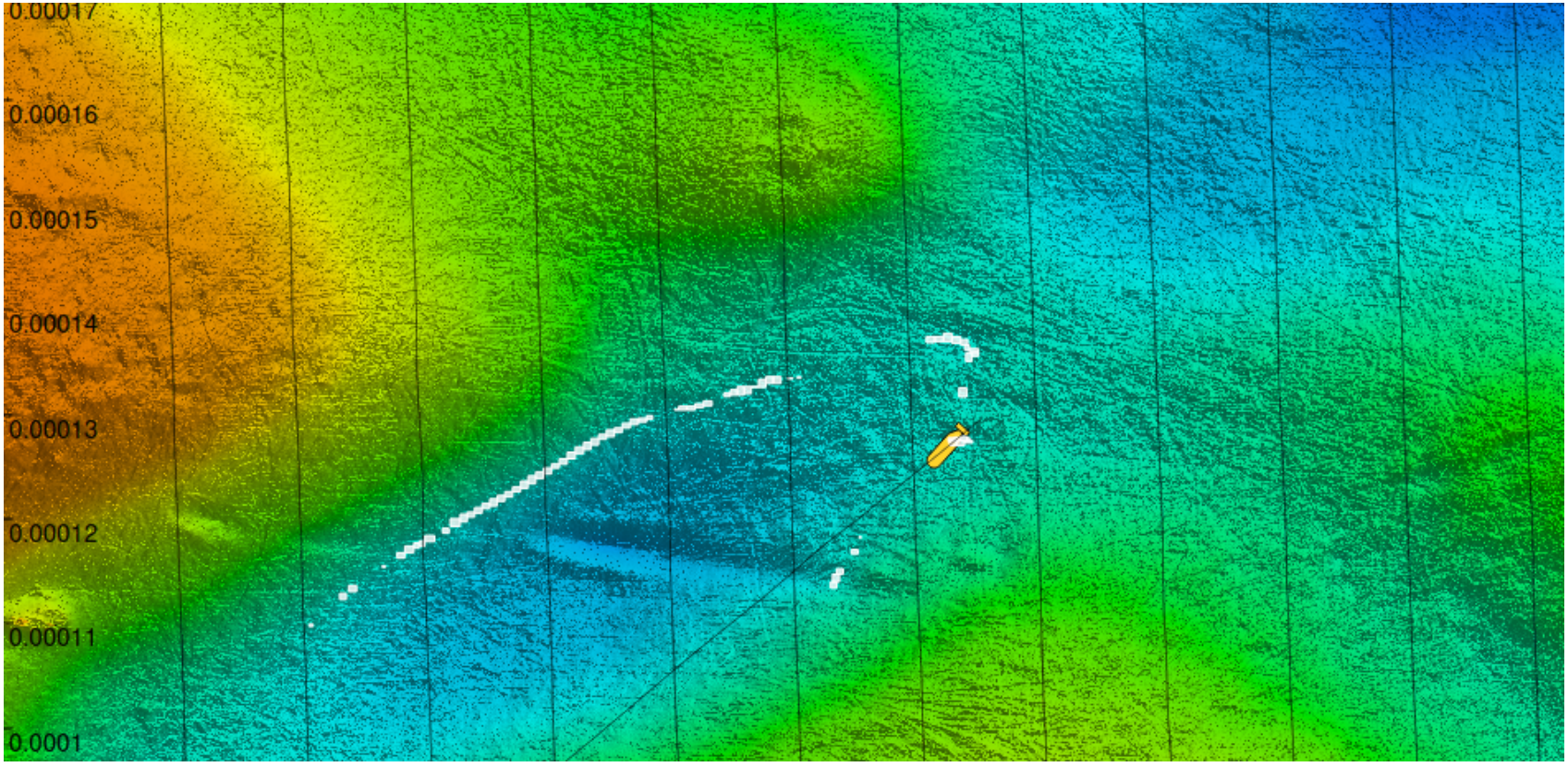
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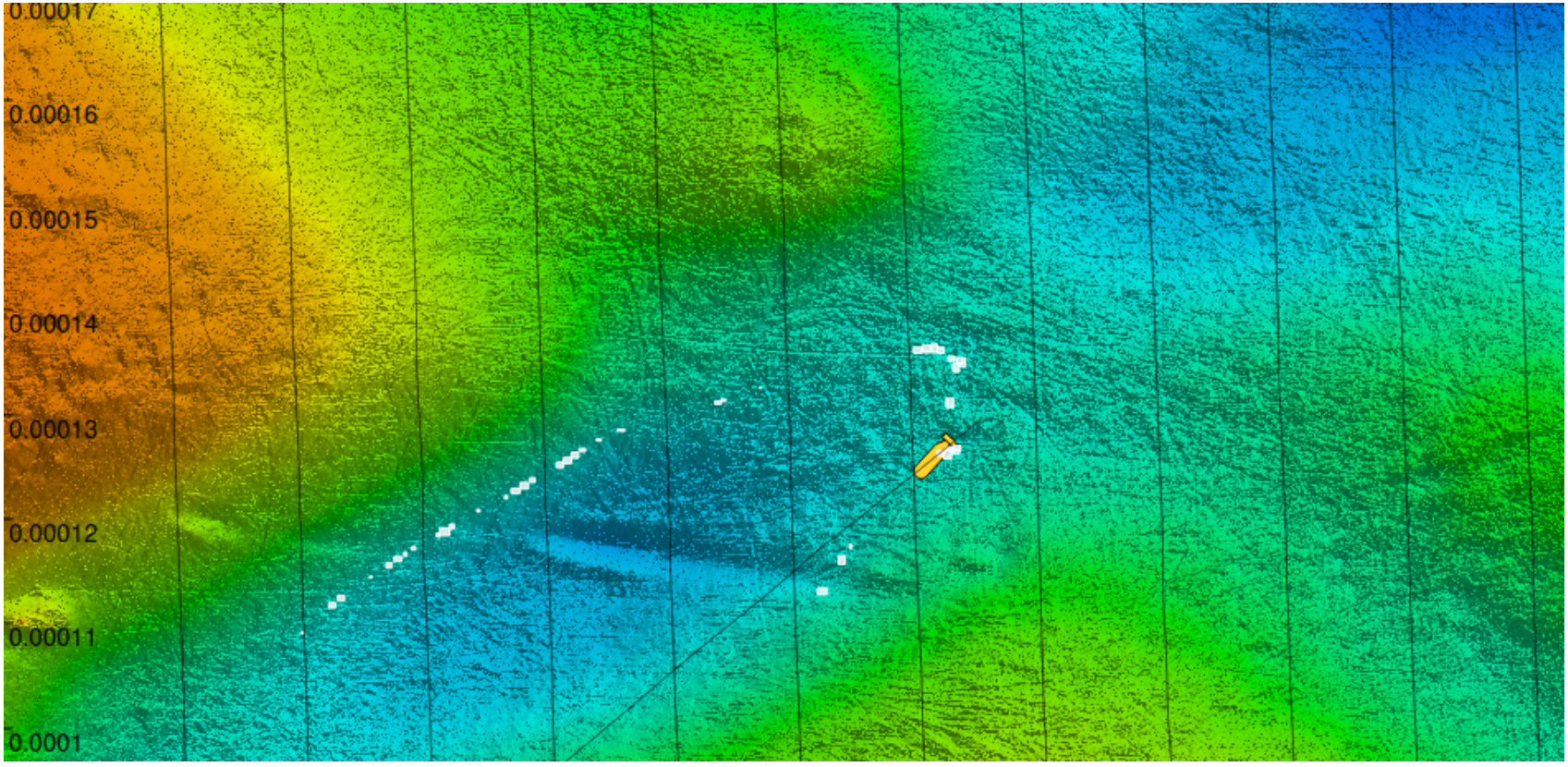
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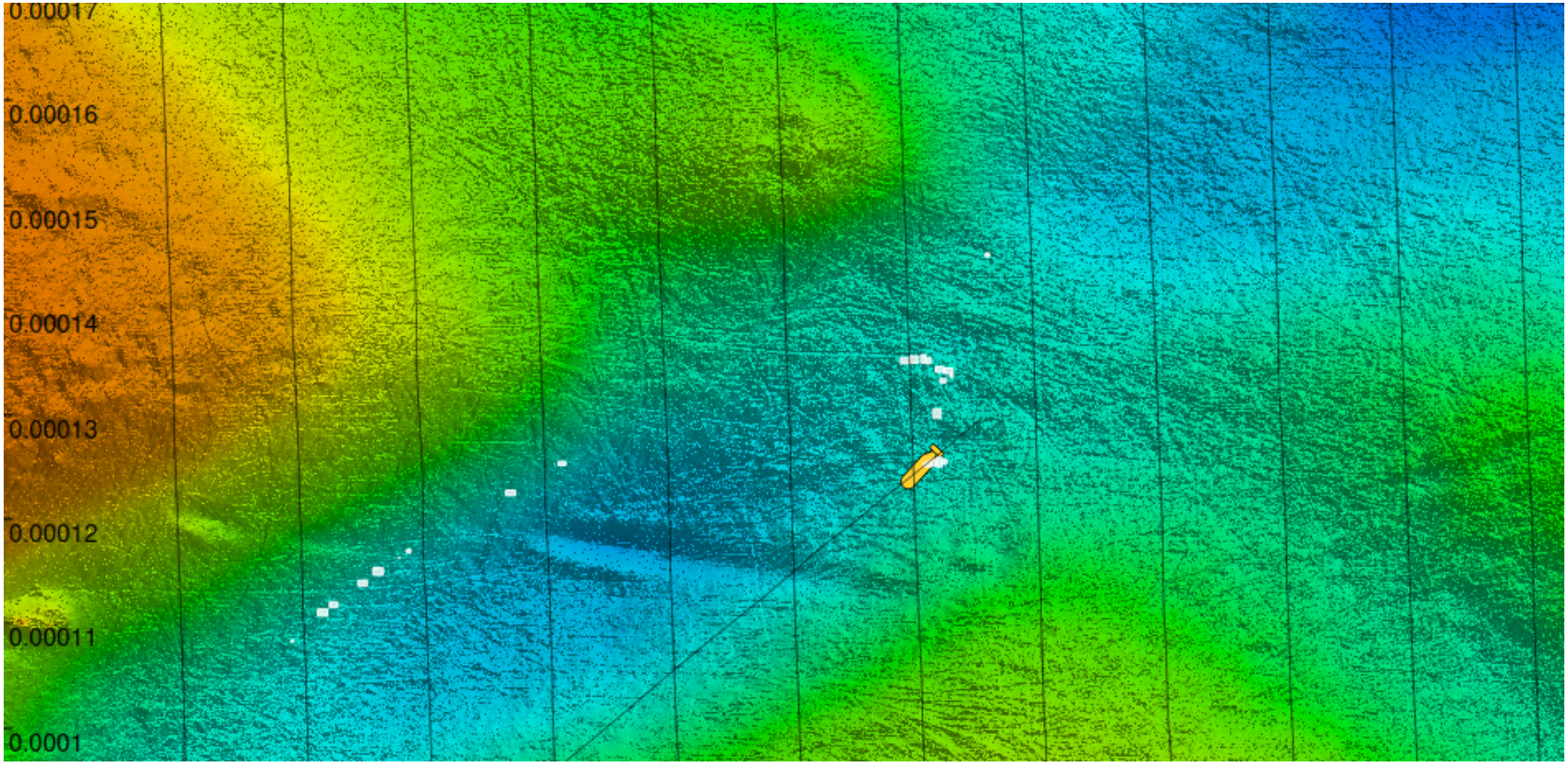
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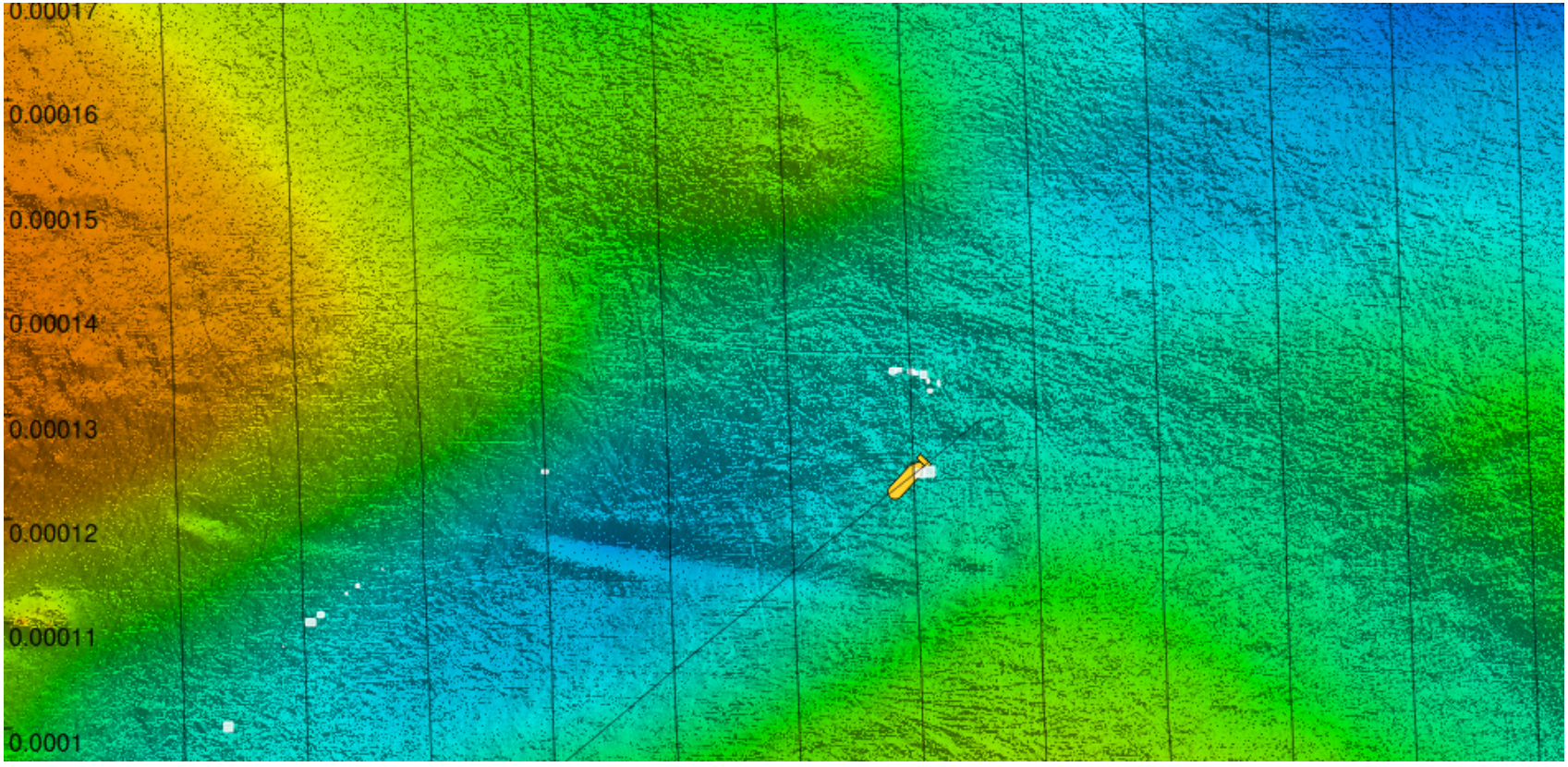
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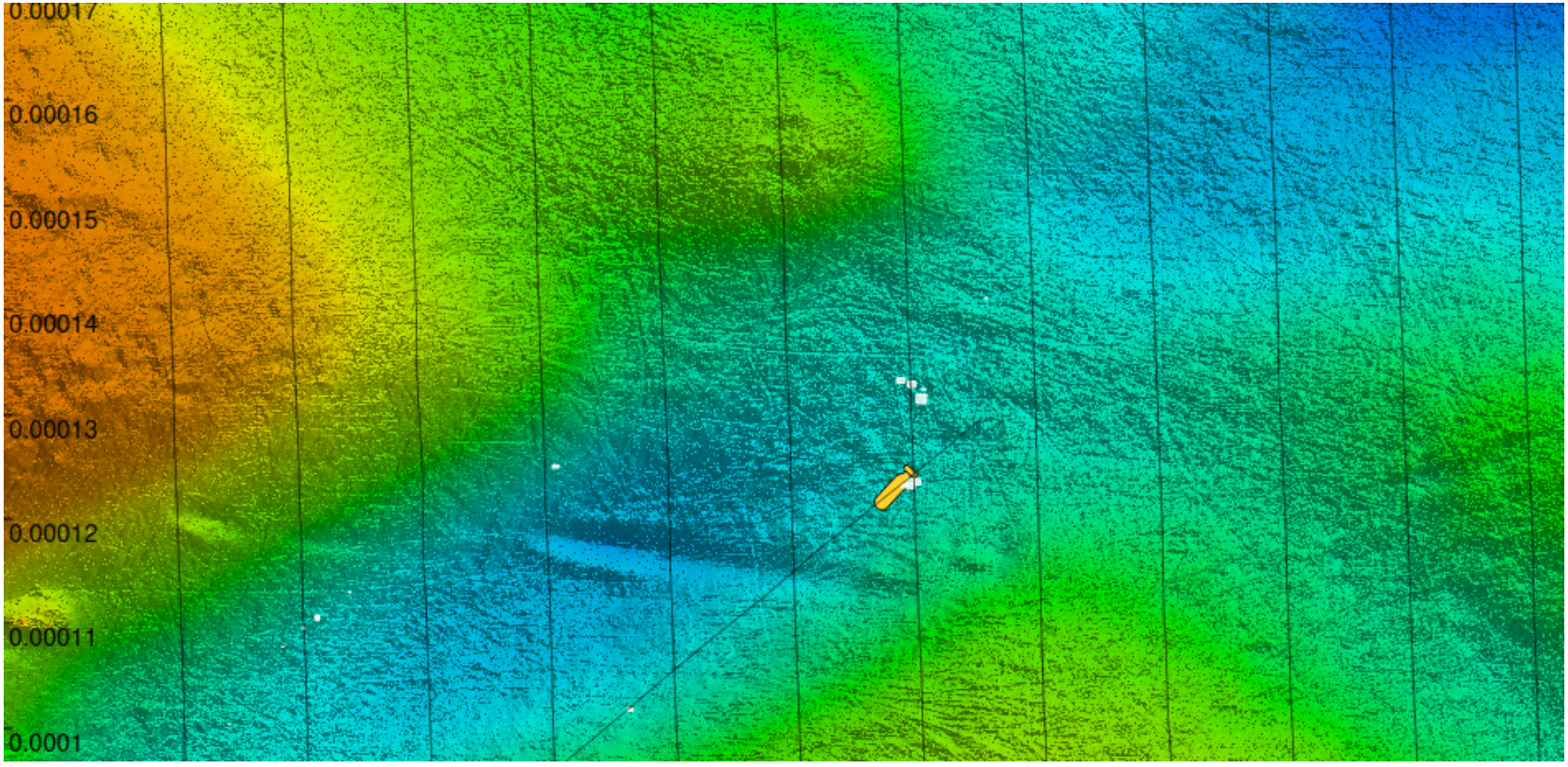
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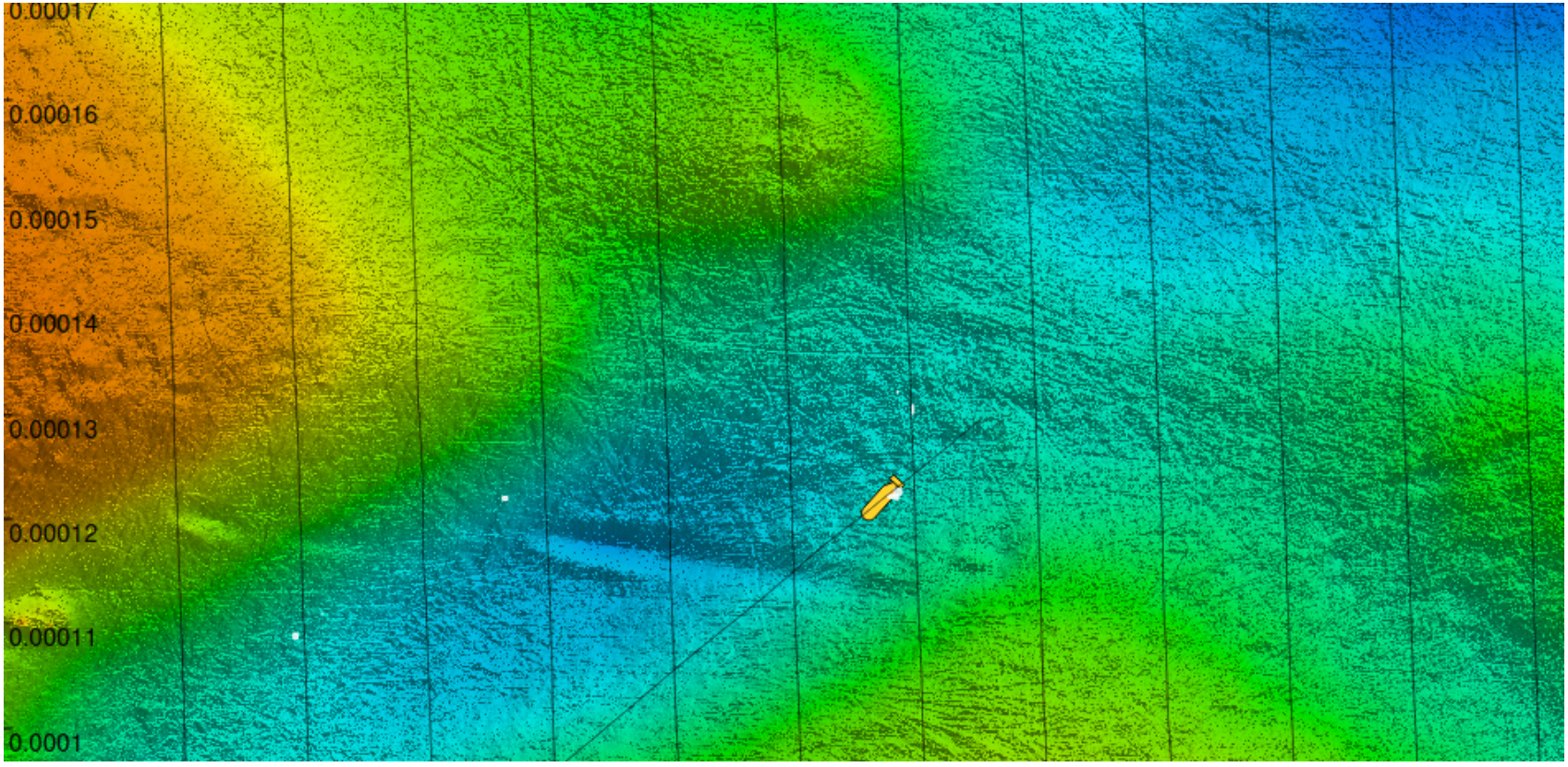
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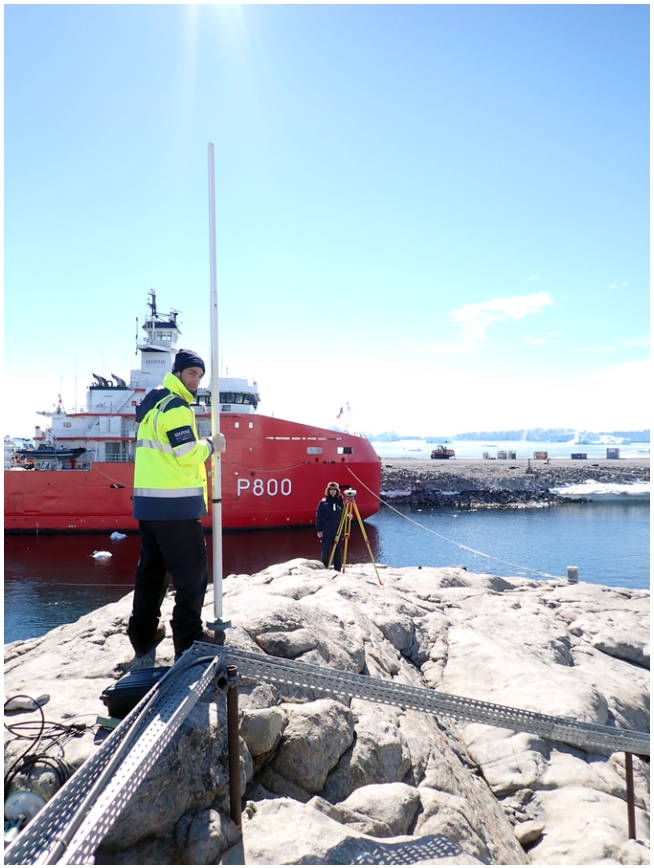
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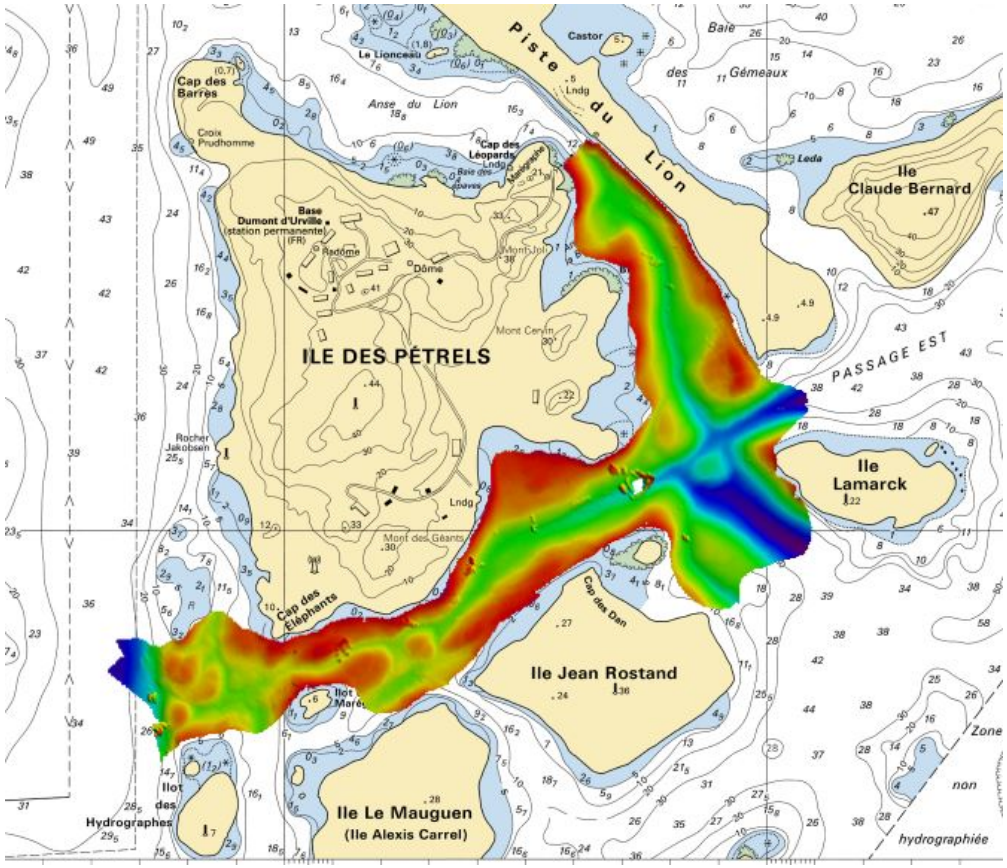
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What about Digital Elevation Models (DEM) in Terre Adélie?

Recent hydrographic survey in Adélie Land conducted by SHOM:





https://www.shom.fr/fr/liste-actualites/mission-hydrographique-en-terre-adelie

Heading observation from position constraints

Evolution equation $\mathbf{f}$ (example of a wheeled robot).

Let us consider the system described by the following equations:

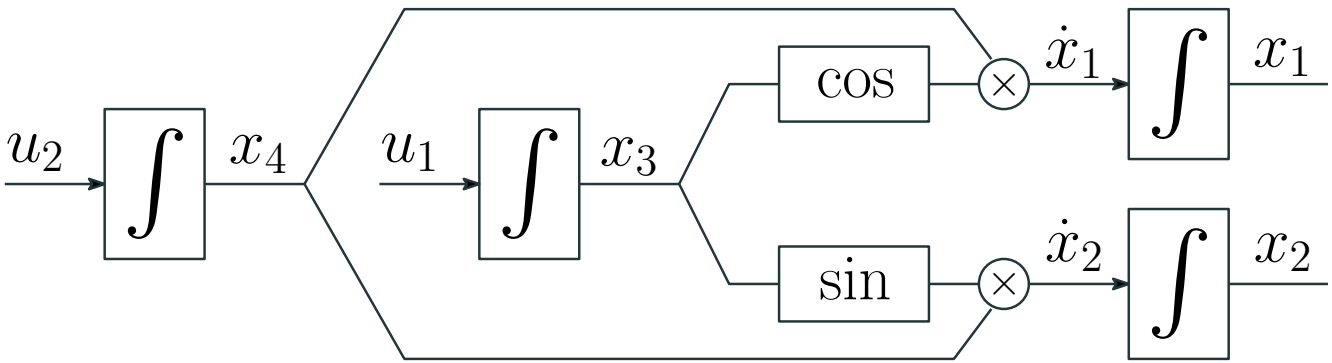
$$\mathbf{f} \begin{cases} \dot{x}_1 &= x_4 \cos(x_3) \\ \dot{x}_2 &= x_4 \sin(x_3) \\ \dot{x}_3 &= u_1 \\ \dot{x}_4 &= u_2 \end{cases} \tag{2}$$

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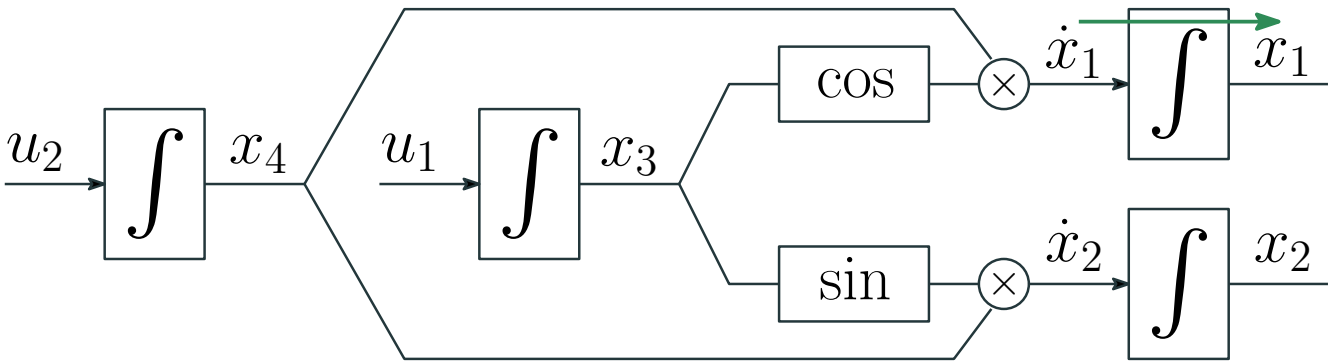


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The problem is difficult when x_3 is unknown.

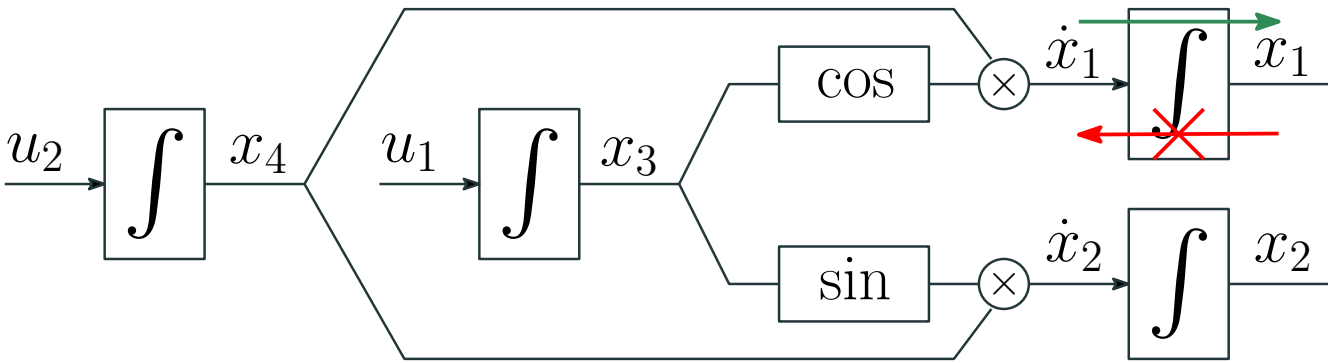
Information only comes from x_1, x_2, $\mathbf{u}$.

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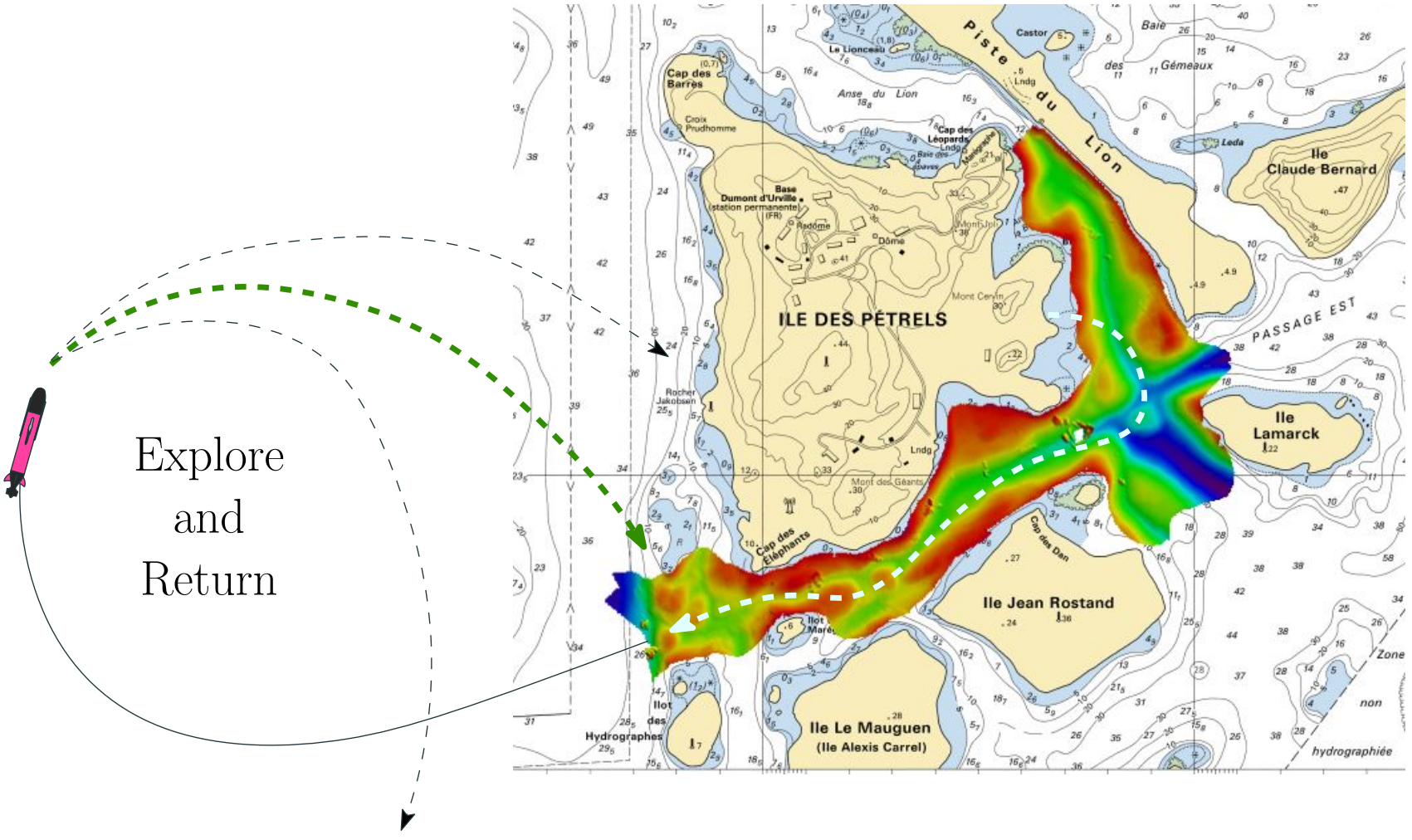
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The Explore and Return problem



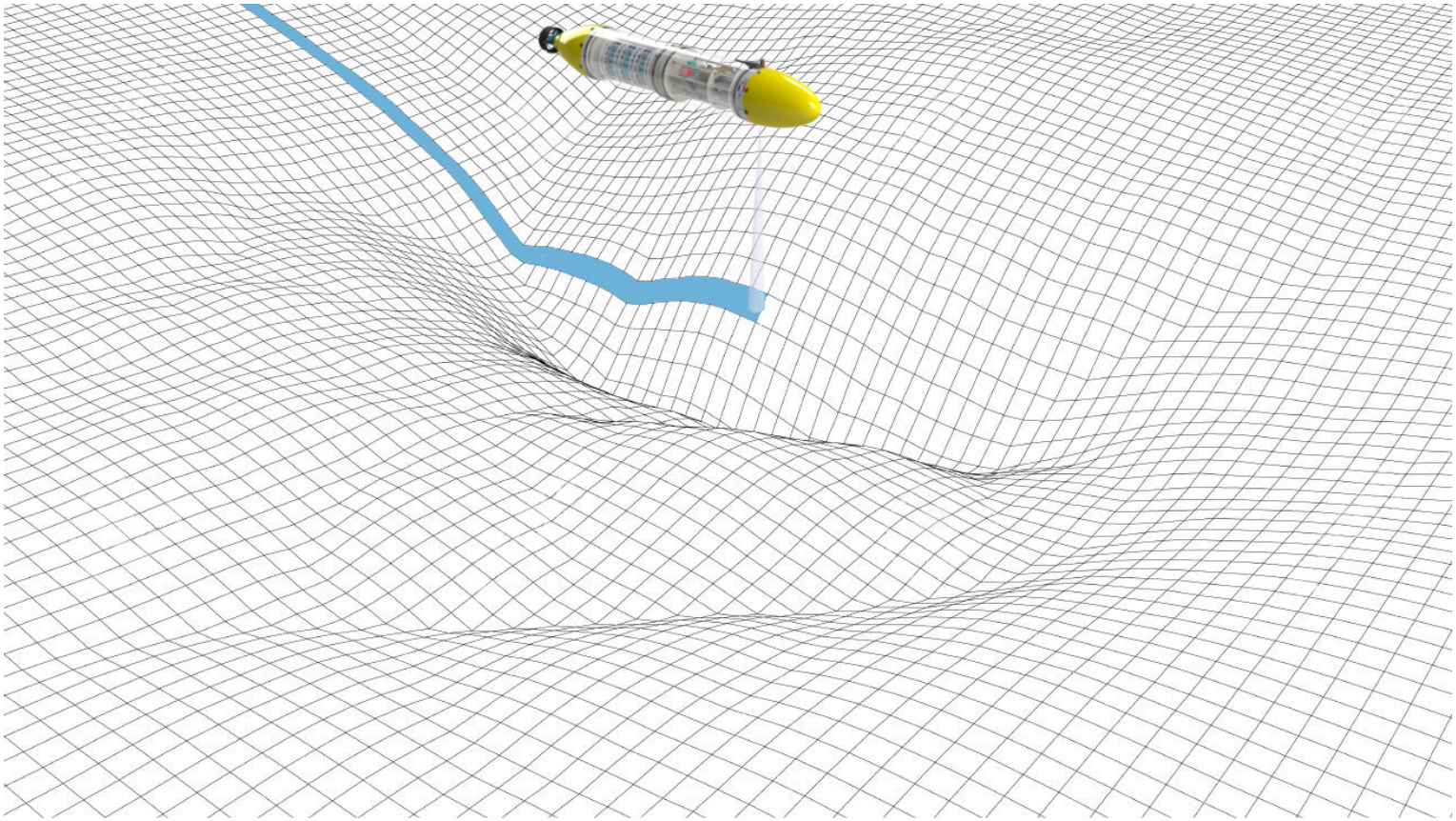
$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{u}(t))$$

$$\mathcal{R} = \{\mathbf{x} \in \mathbb{X} \mid \exists \tau \geq 0,\ \exists \mathbf{u}(\cdot),\ \mathbf{x}(\tau; \mathbf{x}, \mathbf{u}) \in \mathbb{X}_{\text{DEM}}\}$$

Solution 2: Simultaneous Localization And Mapping (SLAM)

And beyond that? SLAM?

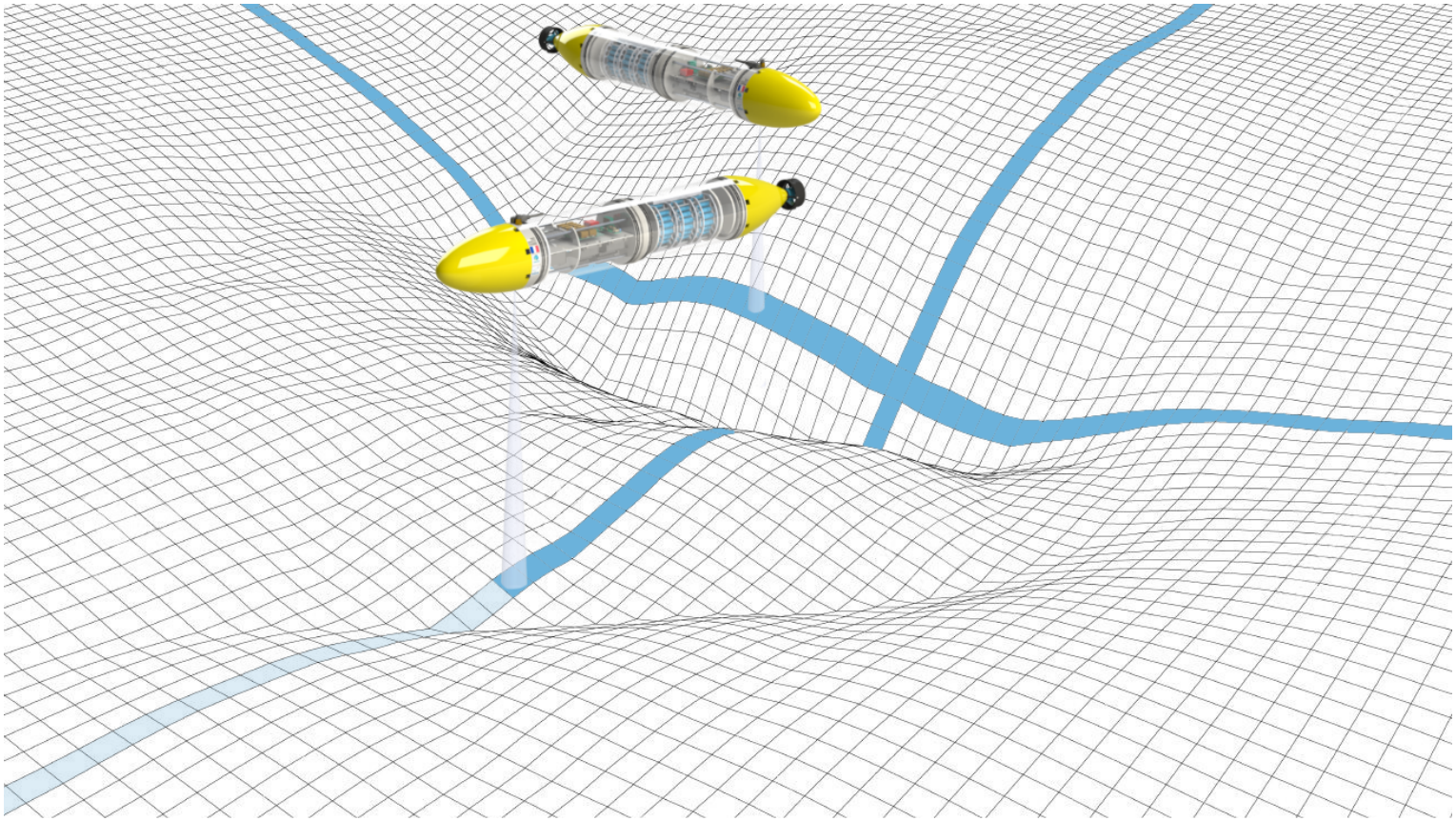
An underwater robot performing a **loop** during an exploration:



Robot's trajectory is projected in blue on the seabed.

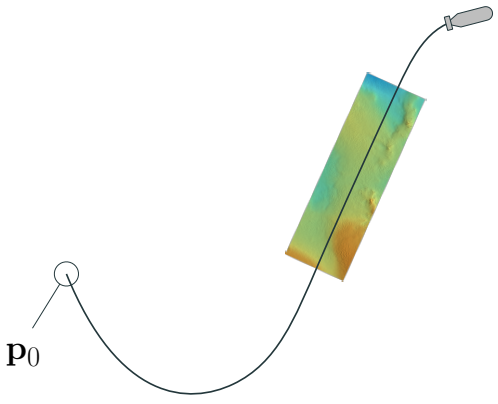
And beyond that? SLAM?

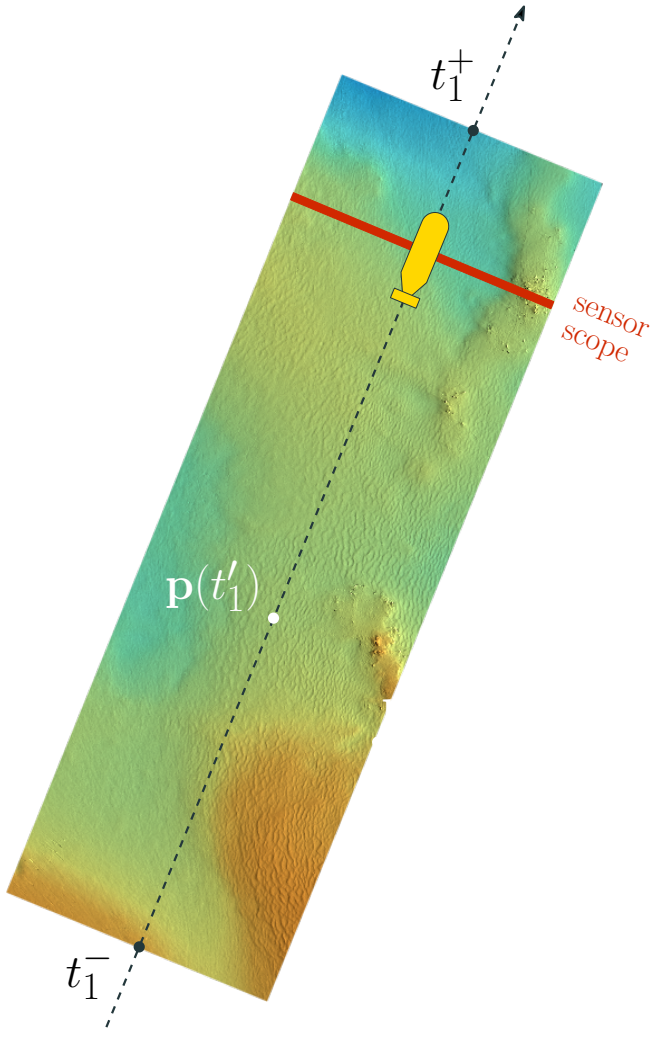
An underwater robot performing a **loop** during an exploration:



Robot's trajectory is projected in blue on the seabed.

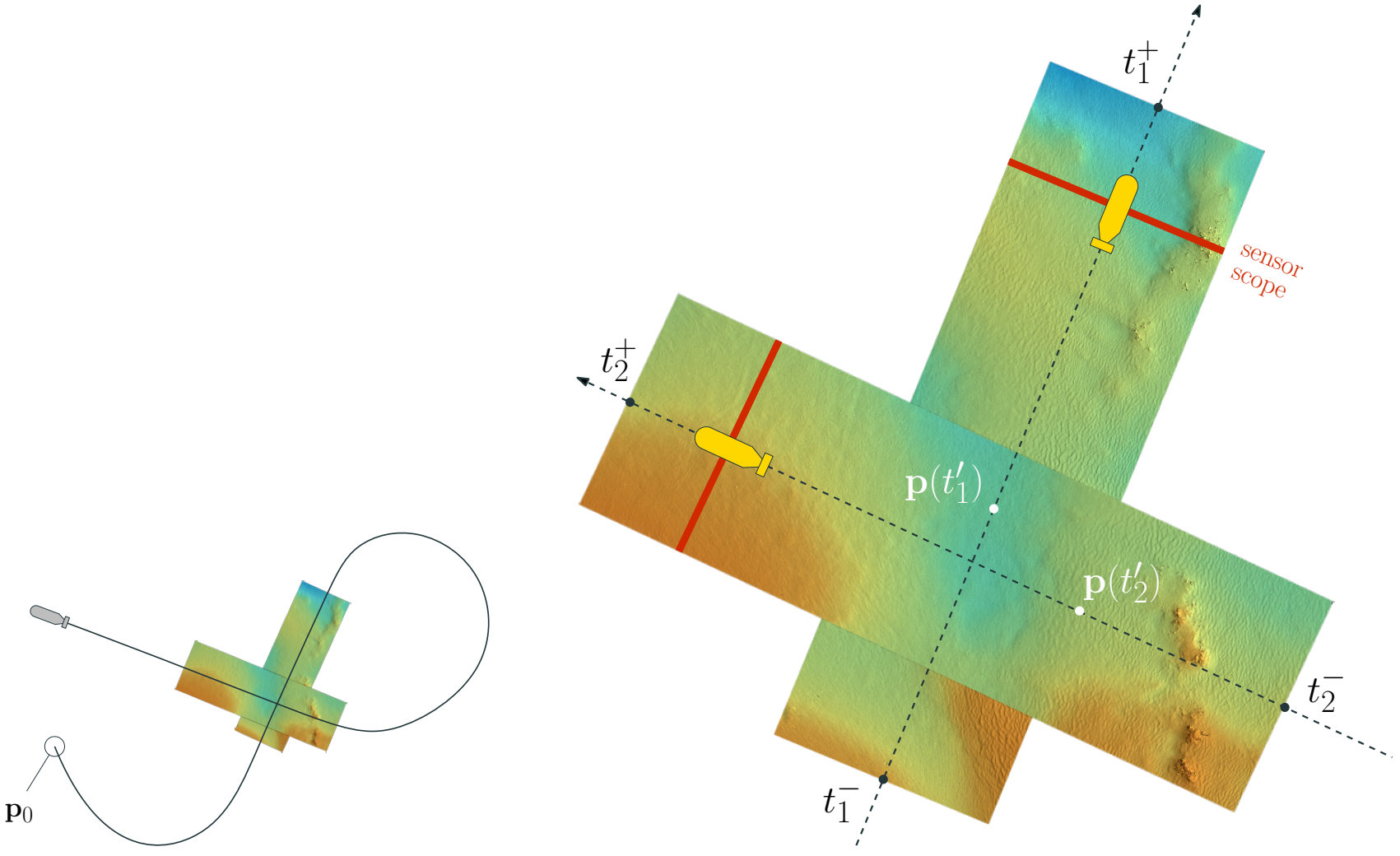
Loop closure with ground perception





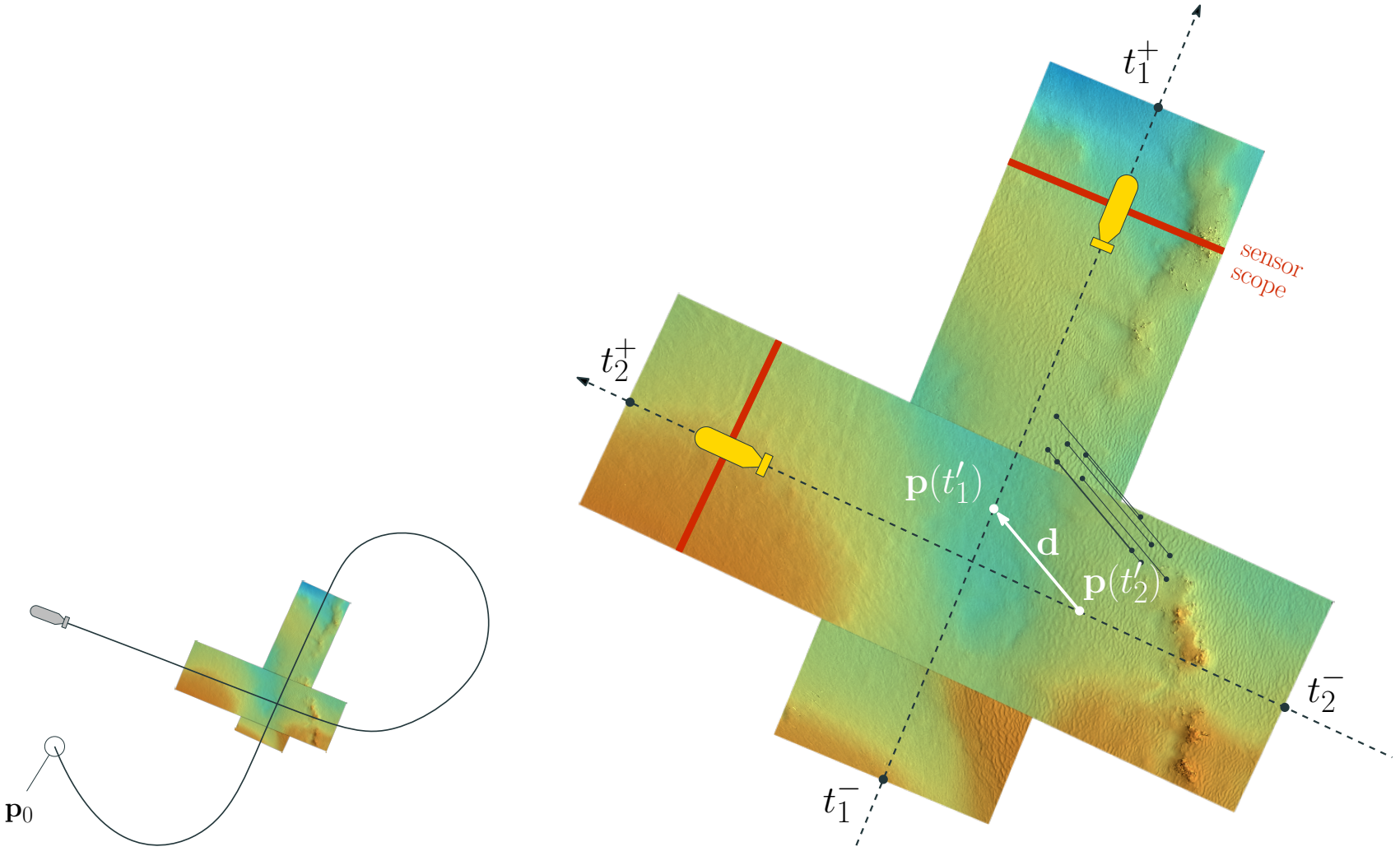
Robot exploring an environment...

Loop closure with ground perception



Robot exploring an environment... coming back to a previous position...

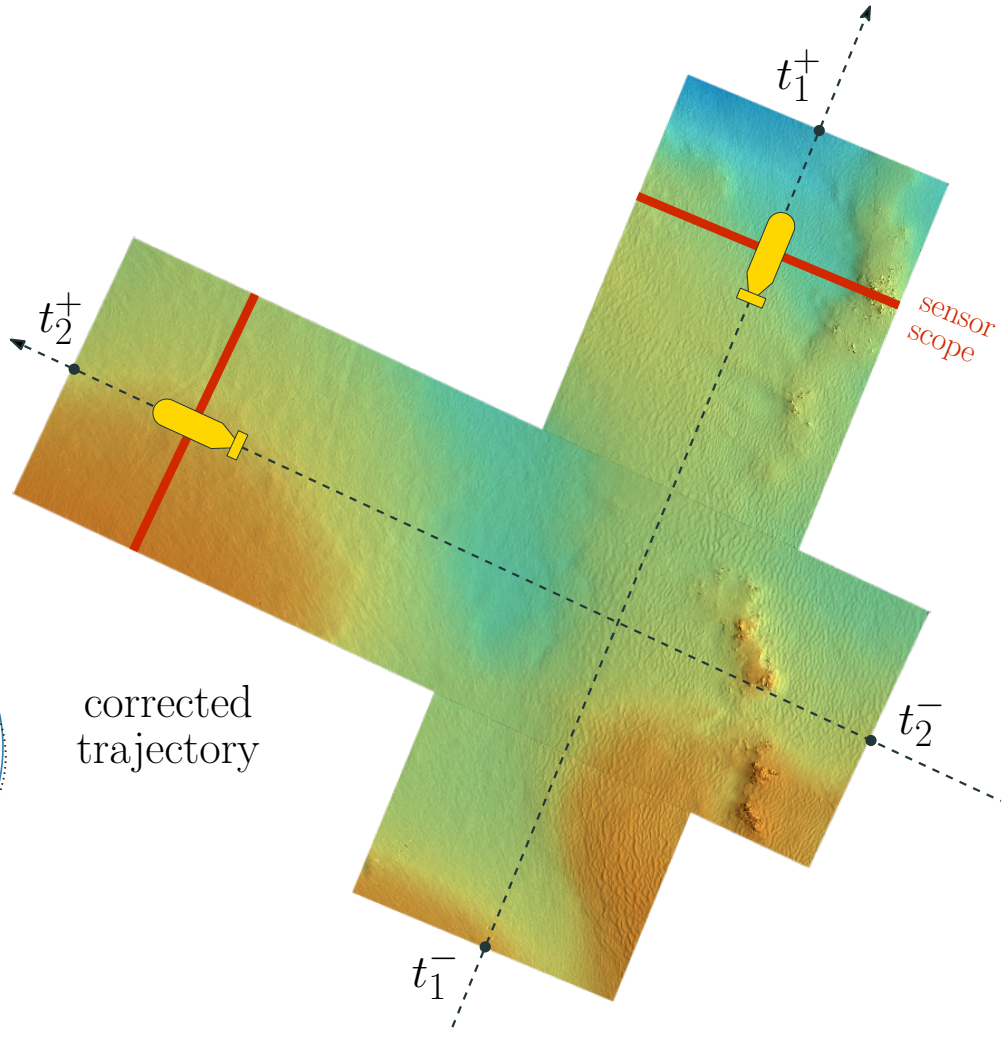
Loop closure with ground perception

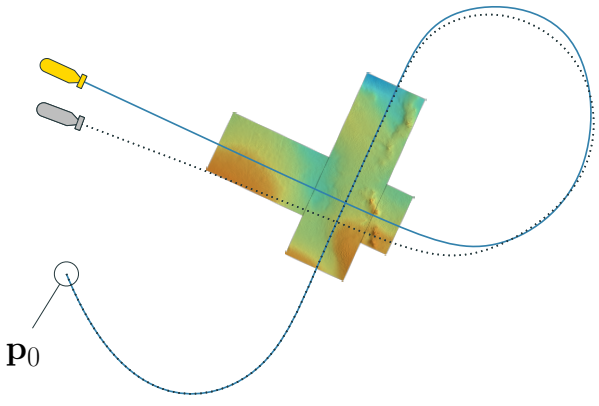


Robot exploring an environment... coming back to a previous position... detecting loop closure...

Loop closure with ground perception

SLAM:
Simultaneous
Localization
And Mapping



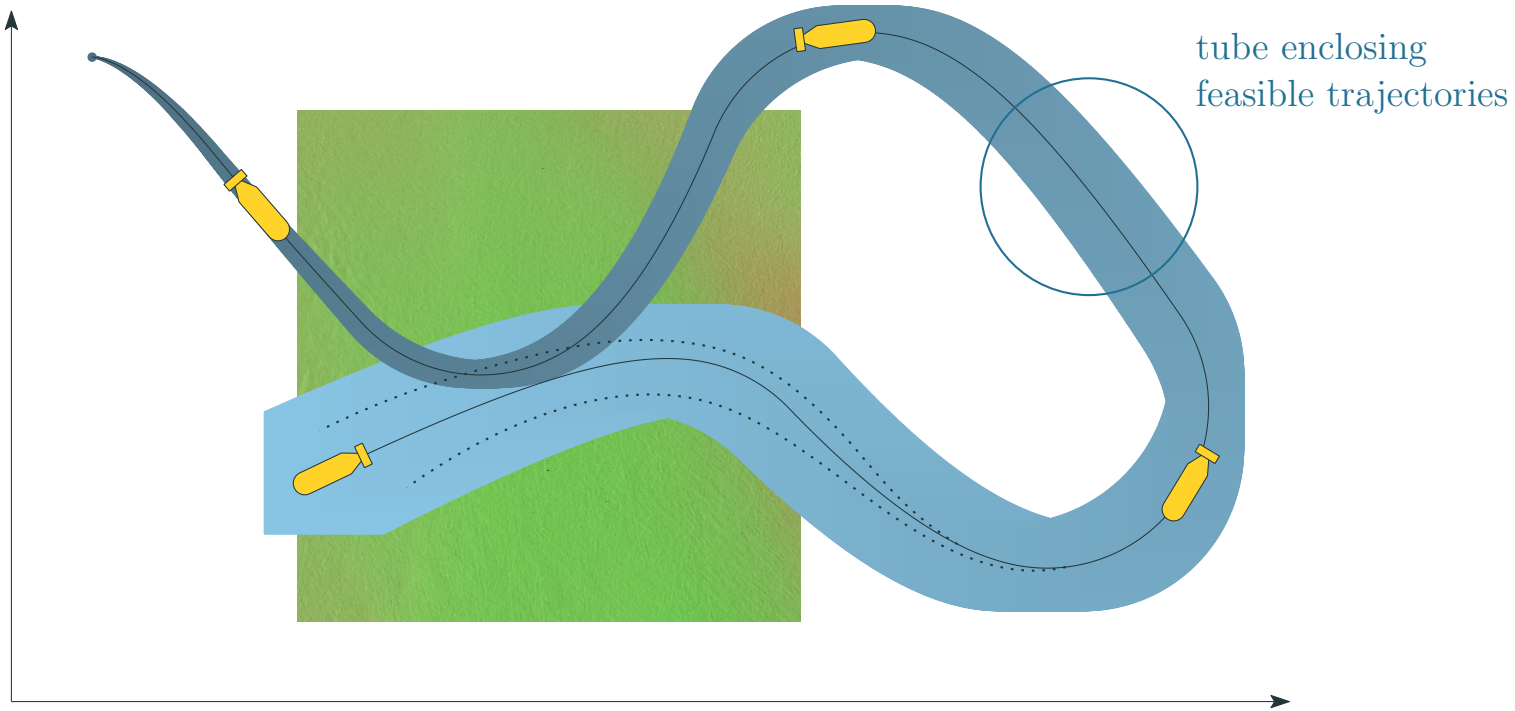


Robot exploring an environment... coming back to a previous position... detecting loop closure... correcting its trajectory.

Problem: doubtful loops and similar environments

What if we recognize a **wrong scene**?

- homogeneous environments $\Longrightarrow$ similar observations
- strong positioning drift $\Longrightarrow$ false loop detections

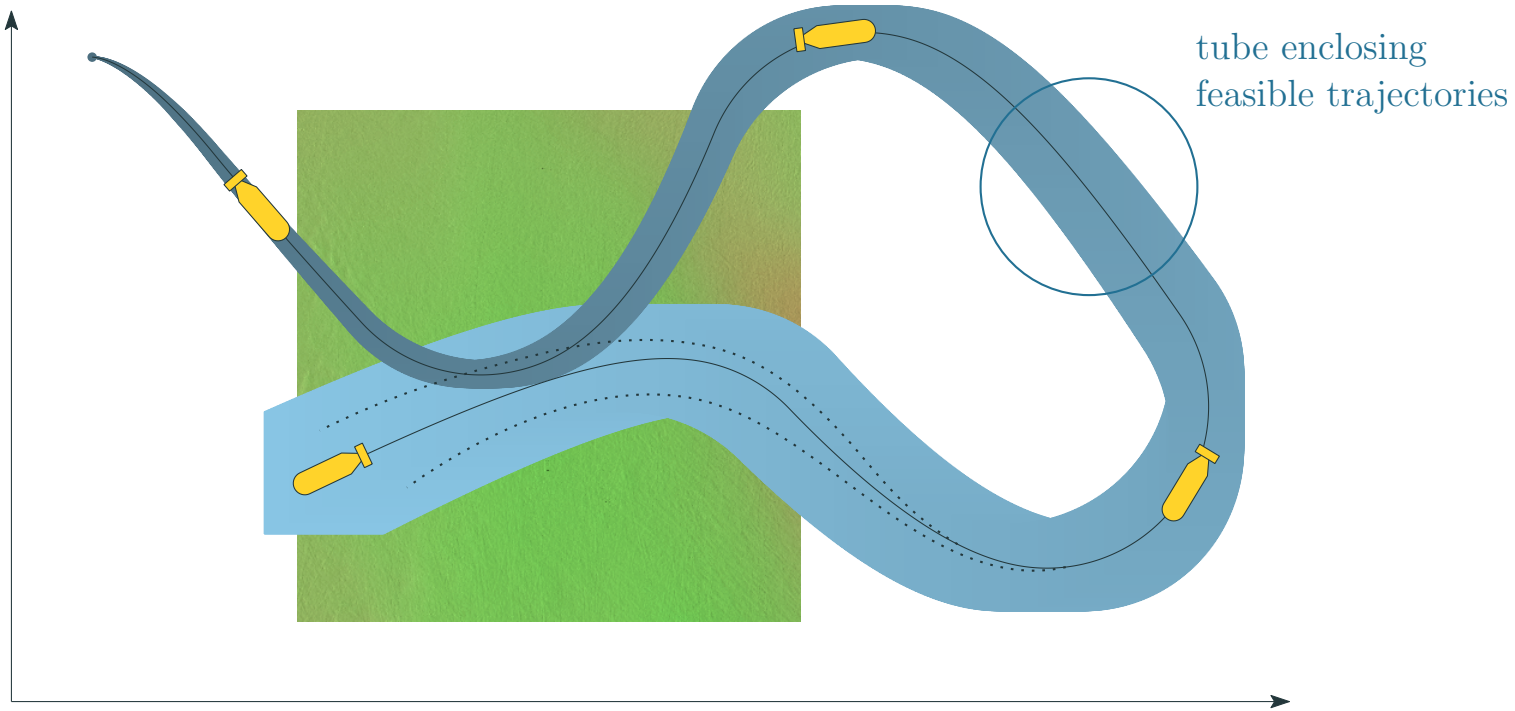


Doubtful loop: the actual trajectory did not loop, but the evolution uncertainties and the homogeneity of the environment may lead to an incorrect correction.

Problem: doubtful loops and similar environments

What if we recognize a **wrong scene**?

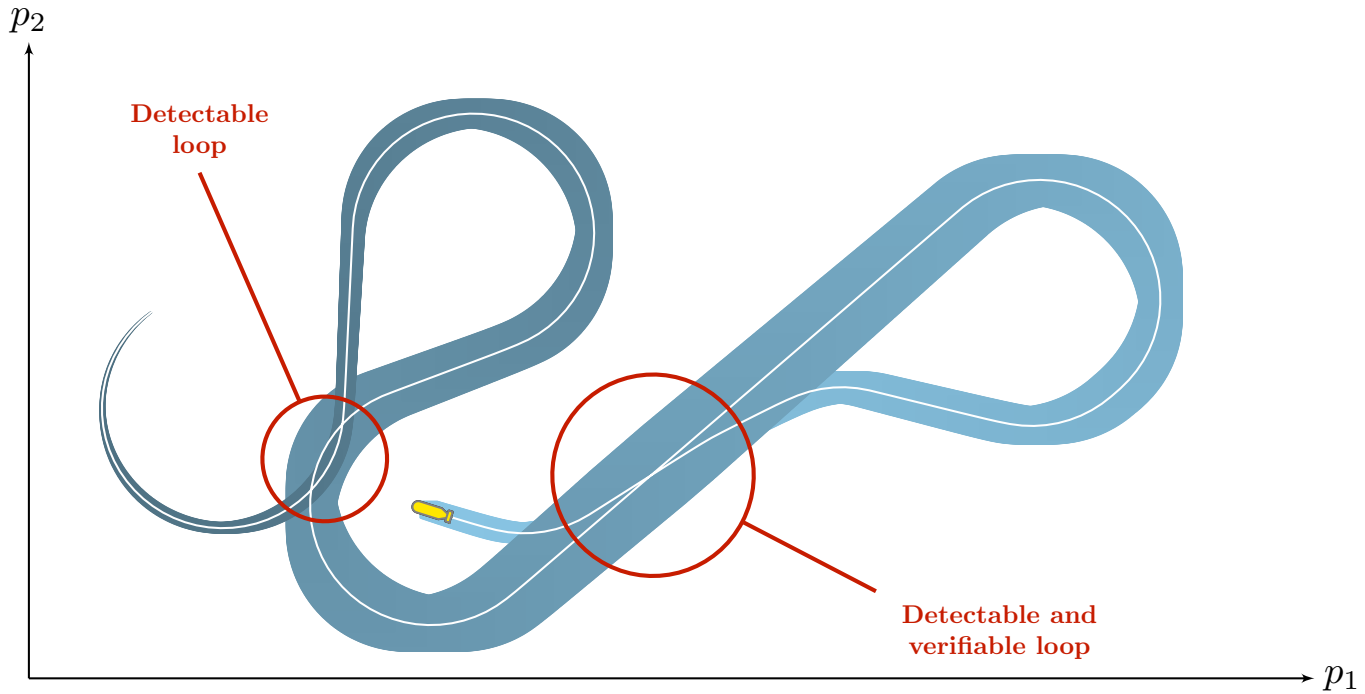
- homogeneous environments $\Longrightarrow$ similar observations
- strong positioning drift $\Longrightarrow$ false loop detections



Doubtful loop: the actual trajectory did not loop, but the evolution uncertainties and the homogeneity of the environment may lead to an incorrect correction. $\Longrightarrow$ *significant impact on the next SLAM iterations.*

Uncertainties: detection vs verification

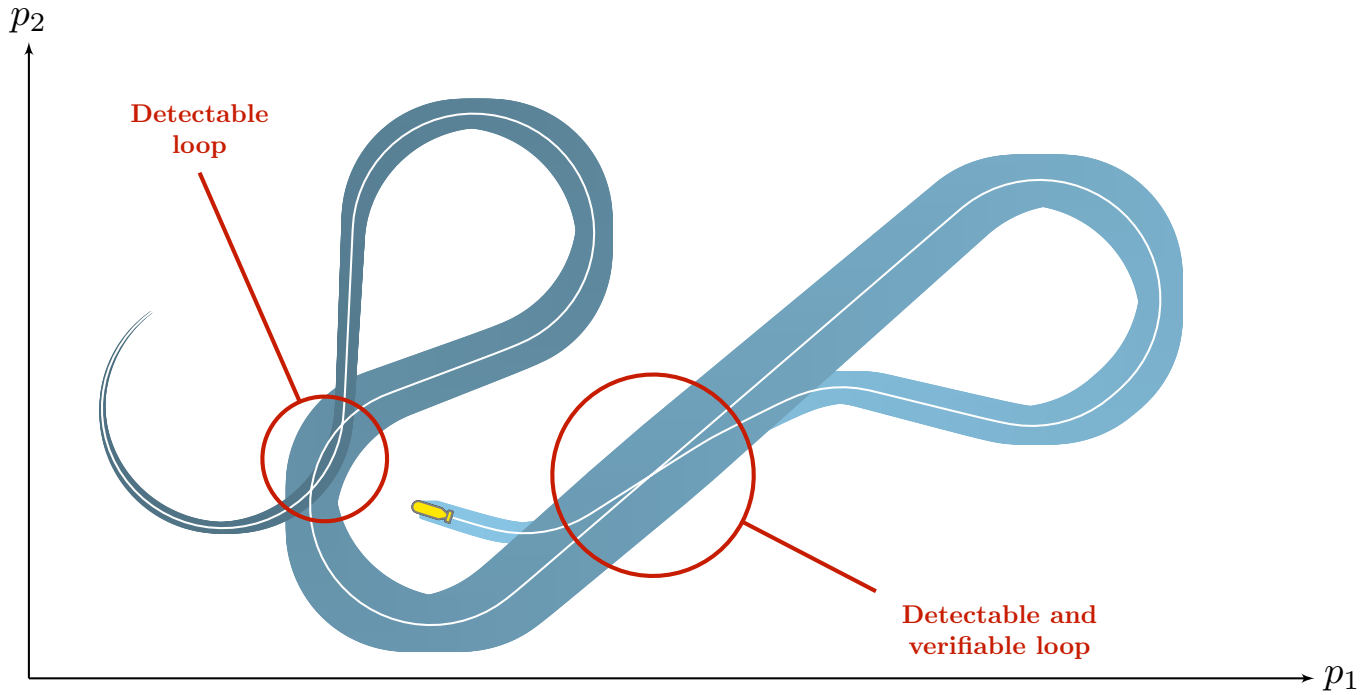
Uncertain trajectories are enclosed by **tubes**.



Only 1 loop can be verified – at least 2 feasible loops are detected.

Uncertainties: detection vs verification

Uncertain trajectories are enclosed by **tubes**.



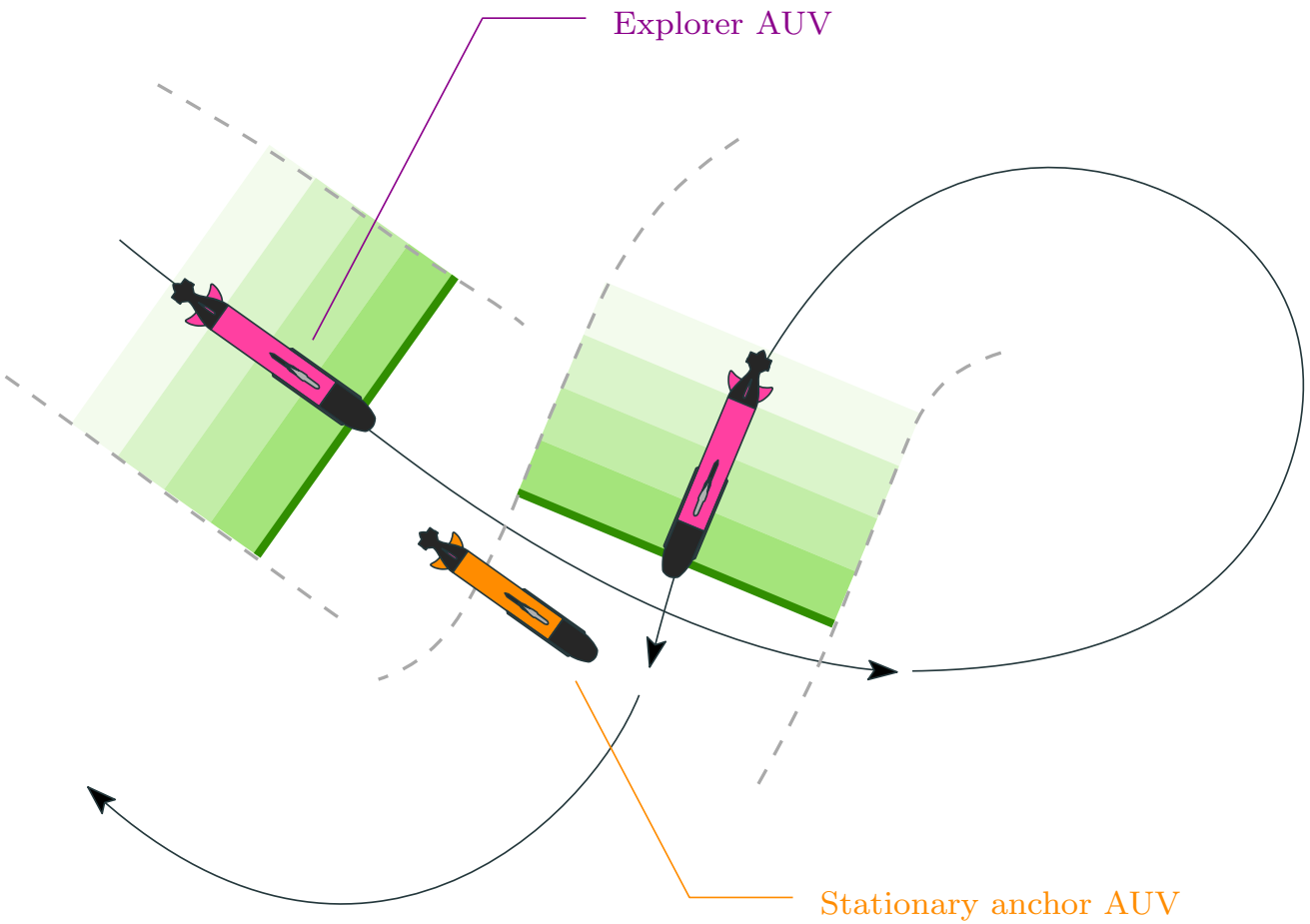
Only 1 loop can be verified – at least 2 feasible loops are detected.

Need for **loop proof**:

- verify that a trajectory crosses itself at some point
- ..whatever the uncertainties describing this trajectory

Compass-free collaborative multi-AUV SLAM

Polar exploration: possible multi-robot approaches.



- distributed systems
- shared uncertainty estimation

Thank you!